Risk management in natural resources management

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Introduction

Since 1947, when GB Dantzig proposed the Simplex algorithm[1], Mathematical Optimization has become one of the most powerful tools for optimal resource planning.

The general formulation of a linear problem is:

$$\begin{aligned} \text{mín} \quad & \mathbf{c}^t\mathbf{x} \\ \text{s.a.} \quad & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

where $\mathbf{c}, \mathbf{A}, \mathbf{b}$ are known.

An optimal solution $\mathbf{x}^*$ is a feasible solution such that $\mathbf{c}^T\mathbf{x} \geq \mathbf{c}^T\mathbf{x}^*$ for all feasible solutions.

[1] One of the top10 influential algorithms from the last century

Introduction

The field of Stochastic Programming (SP) appears as a response to the need of incorporating uncertainty in mathematical models.

SP deals with mathematical programs in which some parameters are random variables.

Early work started in 1955 with Dantzig and Beale. Their methods involve an action followed by observation and reaction or recourse.

Charnes and Cooper in 1959 developed an alternative model called Chance or Probabilistically Constrained Programming.

Even though both methods have their roots in statistical decision theory (Wald, 1950), SP focuses on methods of solution and analytical properties instead of constructing derivatives and updating probabilities.

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Stochastic Linear Models. Decisions and stages

Stochastic Linear Models are linear models is which some parameters are unknown.

These parameters are represented as random variables (random experiment).

$$\begin{array}{rc} \text{mín} & \mathbf{c}(\xi)^t\mathbf{x} \\ \text{s.a.} & \mathbf{A}(\xi)\mathbf{x} = \mathbf{b}(\xi) \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

A recursion problem is a problem in which some decisions have to be taken under uncertainty and some others once the uncertainty has disappeared.

Stochastic Linear Models. Decisions and stages

Therefore, decisions can be classified as:

- First stage decision, represented by $\overline{\mathbf{x}}$, are taken before the result of the random experiment.
- Second stage decisions, represented by $\overline{\mathbf{y}}(\omega)$ or $\overline{\mathbf{y}}(\omega, \overline{\mathbf{x}})$, have to be taken once the random experiment is known.

Therefore, the sequence of events and decision is

$$\mathbf{x} \to \xi(\omega) \to \mathbf{y}(\omega, \overline{\mathbf{x}})$$

Two-stage Stochastic Models (Beale, 1955 y Dantzig, 1955)

$$
\begin{array}{llll}
\text{mín} & z = \mathbf{c}^T\mathbf{x} + E_\xi\left[\text{mín}\,\mathbf{q}^{\omega\,T}\mathbf{y}^\omega\right] & & \\
\text{s. a} & \mathbf{A}\mathbf{x} & = \mathbf{b} & \\
 & \mathbf{T}^\omega\mathbf{x} \qquad + \mathbf{W}^\omega\mathbf{y}^\omega & = \mathbf{h}^\omega & \forall\omega\in\Omega \\
 & \mathbf{x}, \qquad \mathbf{y}^\omega & \geq \mathbf{0} & \forall\omega\in\Omega
\end{array}
$$

- $\mathbf{x}$ is the vector of first stage decision variables,
- ξ takes values depending on $\omega \in \Omega$.
- For each realization ω of ξ, $\mathbf{q}^\omega$, $\mathbf{h}^\omega$, $\mathbf{W}^\omega$ and $\mathbf{T}^\omega$ are known.
- $\mathbf{y}^\omega$ is the vector of second stage decision variables,
- $\mathbf{W}^\omega$ is the recourse matrix (if $\mathbf{W}^\omega = \mathbf{W}$, it is fixed recourse problem).

DEM: Deterministic Equivalent Model

For each **x** and ξ, the second-stage problem is:

$$Q(\mathbf{x},\xi(\omega)) = \min_{\mathbf{y}}\left\{{\mathbf{q}^{\omega}}^{T}\mathbf{y} \,/\, \mathbf{W}^{\omega}\mathbf{y} = \mathbf{h}^{\omega} - \mathbf{T}^{\omega}\mathbf{x},\ \mathbf{y} \geq \mathbf{0}\right\}$$

The expected value $\mathcal{Q}$ is:

$$\mathcal{Q}(\mathbf{x}) = E_{\xi}\left[Q(\mathbf{x},\xi(\omega))\right]$$

Therefore, the Two-stage Stochastic Model can be represented by the Deterministic Equivalent Model or Recourse Problem:

$$\begin{aligned} &\min_{\mathbf{x}}\ \mathbf{c}^{T}\mathbf{x} + \mathcal{Q}(\mathbf{x}) \\ &\text{s.a} \qquad \mathbf{A}\mathbf{x} = \mathbf{b} \\ &\qquad\qquad \mathbf{x} \geq \mathbf{0} \end{aligned}$$

Scenario analysis

ξ represents the uncertainty and takes values depending on $\omega \in \Omega$.

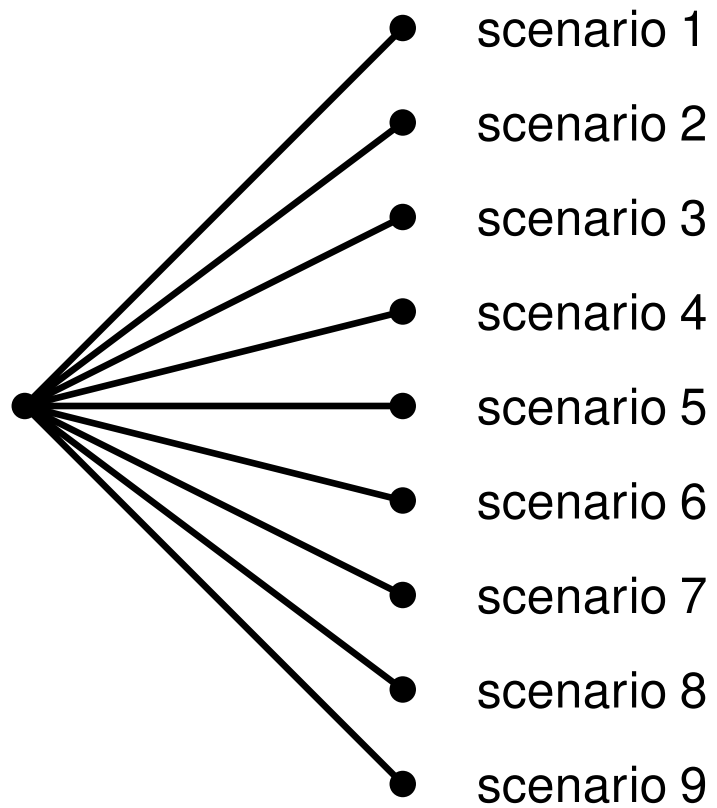
In real-life application, Ω is considered finite.

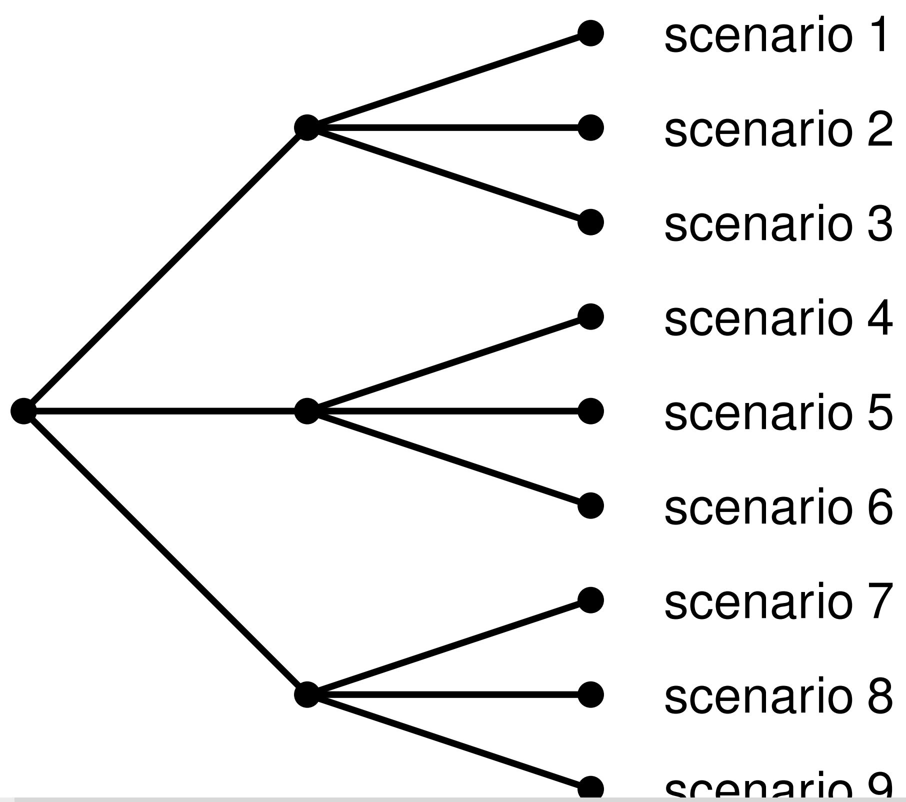
Each element $\omega \in \Omega$ is called scenario and represents a realization of the random variable ξ.

Scenarios are represented in a tree.

Each node in the tree represents the time events at which decisions can be taken.

Each scenario ω has a weight, p^{ω}, that represents the likelihood assigned to this scenario by the decisor.





Non-anticipativity principle

The decision recommended by the model must satisfy the non-anticipativity condition that guarantees the independence of the solutions regarding the information not yet available.

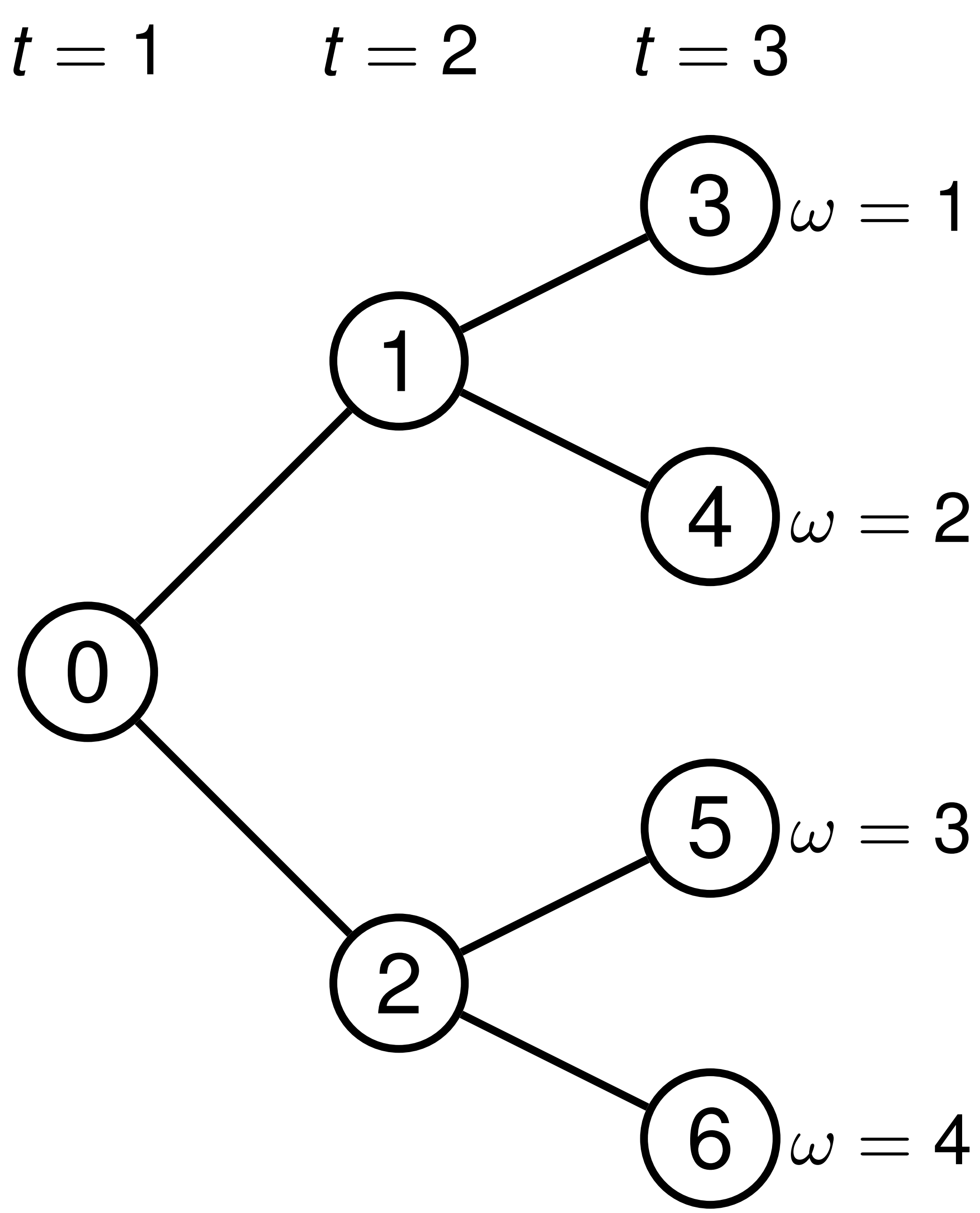
The non-anticipativity principle (Rockafellar and Wets, 1991) says that

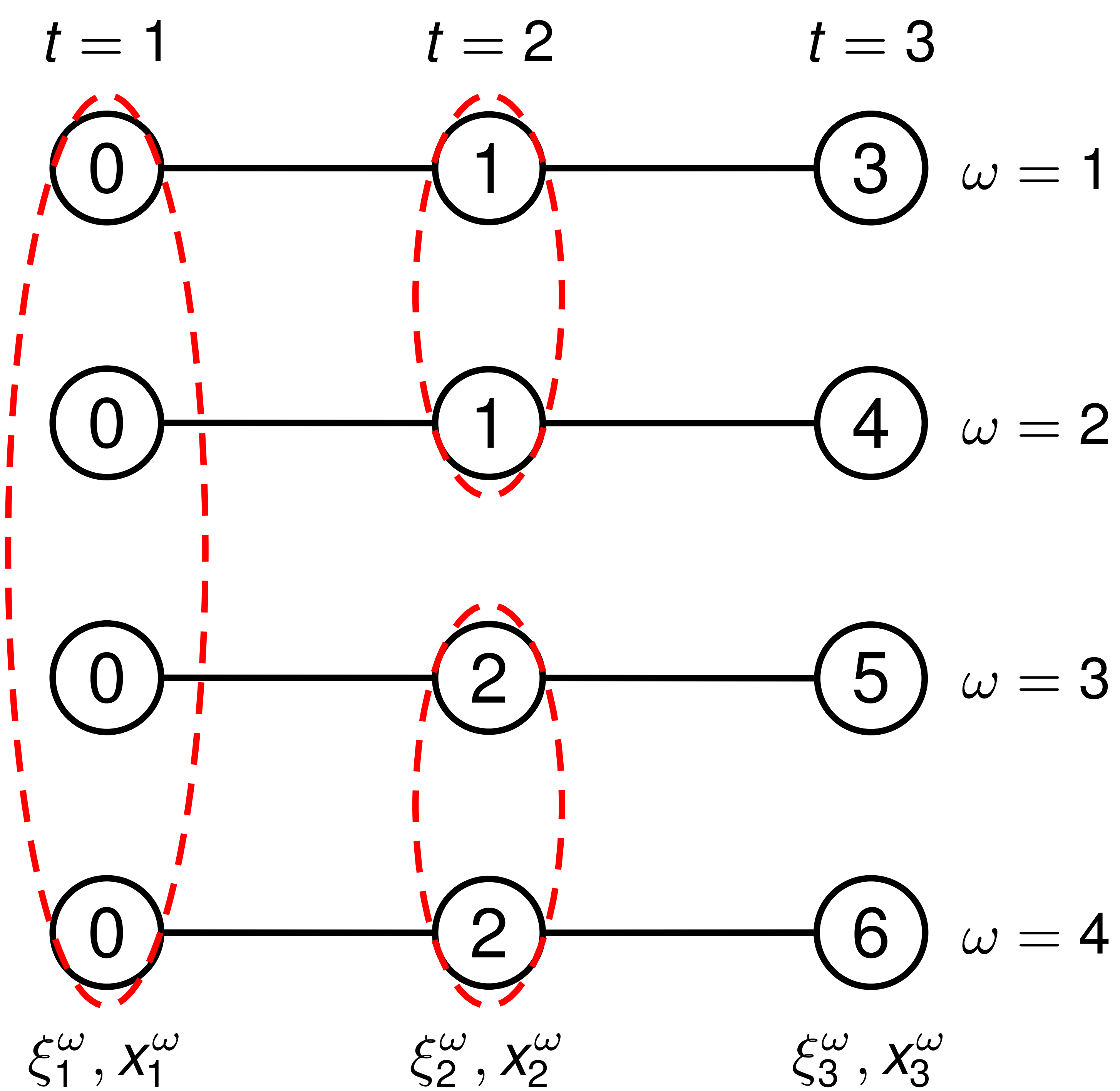
> *"if two different scenarios ω and ω' are identical until stage t about the disponible information in that stage, then the decisions in both scenarios must be the same too until stage t.*

The non-anticipativity principle requires that

$$\mathbf{x}_t^{\omega} = \mathbf{x}_t^{\omega'} \quad \text{if} \quad \xi_\tau^{\omega} = \xi_\tau^{\omega'}, \forall \tau = 1, \ldots, t,$$

Non-anticipativity principle





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Farmer example

Introduction to Stochastic Programming, Birge and Louveaux, 1997

A farmer, specialized in raising grain, corn and sugar beets, has 500Ha. of land.

The farmer needs al least 200 tons (T) of wheat and 240 T of corn for cattle feed.

- This amount can be raised on the farm or bought from a wholesaler.
- Any production in excess of the feeding requirement would be sold.

The production of sugar beet can be sold at 36 € /T.

- However, the European Comission imposes a quota of 6000T, and,
- Any amount in excess of the quota ca be sold only at 10 € /T

Farmer example

Objective: maximization of the profit.

Decisions: assign land to each product.

	Wheat	Corn	Sugar Beets
Yield (T/Ha)	2,5	3	20
Planting cost (€/Ha)	150	230	260
Selling price (€/Ha)	170	150	36 (under 6000T)
			10 (above 6000T)
Purchase price (€/T)	238	210	-
Minimum requeriment	200	240	-
Total available land:	500 Ha		

Farmer example. Deterministic model

Variables

x_i : Ha of land devoted to wheat ($i = 1$), corn ($i = 2$), and sugar beets ($i = 3$).

y_i : tons of wheat ($i = 1$) and corn ($i = 2$) purchased.

z_i : tons of wheat ($i = 1$) and corn ($i = 2$) sold.

z_i : tons of sugar beets sold at the favorable price ($i = 3$), and at the lower price ($i = 4$).

Farmer example. Deterministic model

Formulation

$$
\begin{array}{llllllllll}
\text{mín} & 150x_1 + 230x_2 + 260x_3 + 238y_1 - 170z_1 + 210y_2 - 150z_2 - 36z_3 - 10z_4 \\
\text{s.a.} & x_1 + x_2 + x_3 \leq 500 \\
 & 2{,}5x_1 + y_1 - z_1 \geq 200 \\
 & 3x_2 + y_2 - z_2 \geq 240 \\
 & -20x_3 + z_3 + z_4 \leq 0 \\
 & z_3 \leq 6000 \\
 & x_1, x_2, x_3, y_1, z_1, y_2, z_2, z_3, z_4 \geq 0
\end{array}
$$

Farmer example. Deterministic model

Solution

	Wheat	Corn	Sugar Beets
Surface (Ha)	120	80	300
Yield (T)	300	240	6000
Sales (T)	100	-	6000
Purchase (T)	-	-	-
Overall profit	118600		

Farmer example. Uncertainty

After thinking about this solution, the farmer becomes worried.

He has indeed experienced quite different yields for the same crop over different years, mainly because of changing weather conditions.

Taking into account the weather conditions, yields varying 20 % to 25 % above or below the mean yield are not unusual.

Stochastic Programming allows to introduce this uncertainty into the model, by considering different scenarios.

Farmer example. Uncertainty

We assume some correlation among the yields of the different crops.

we can consider three possible scenarios:

- Good season: yield is 20 % above the average.
- Normal season: yield is equal to the average.
- Bad season: yield is 20 % below the average.

For simplicity, we assume that weather conditions and yields for the farmer do no have a significant impact on prices.

Farmer example

Optimal solution for each scenario

	Above (+20 %)			Average			Below (-20 %)		
	Whea	Corn	Sugar Beets	Whea	Corn	Sugar Beets	Whea	Corn	Sugar Beets
Surface (Ha)	183,33	66,67	250	120	80	300	100	25	375
Yield (T)	550	240	6000	300	240	6000	200	60	6000
Sales (T)	350	-	6000	100	-	6000	-	-	6000
Purchase (T)	-	-	-	-	-	-	-	180	-
Overall profit	167667			118600			59950		

Farmer example

Optimal solution for each scenario. Analysis

	Above (+20 %)			Average			Below (-20 %)		
	Whea	Corn	Sugar Beets	Whea	Cor	Sugar Beets	Whea	Cor	Sugar Beets
Surface (Ha)	183,33	66,67	250	120	80	300	100	25	375
Yield (T)	550	240	6000	300	240	6000	200	60	6000
Sales (T)	350	-	6000	100	-	6000	-	-	6000
Purchase (T)	-	-	-	-	-	-	-	180	-
Overall profit	167667			118600			59950		

The optimal solution is very sensitive to changes in yields:

- The surfaces devoted to wheat range from 100 Ha to 183.33 Ha;
- The surfaces devoted to corn range from 25 Ha to 66.67 Ha; and
- The surfaces devoted to sugar beets range from 250 Ha to 375 Ha.
- The overall profit ranges from 59950 € to 167667 € .

Farmer example. Solution analysis

Therefore, long-term weather forecast would be very helpful here.

Unfortunately, meteorologists agree that weather conditions cannot be accurately predicted six months ahead.

Therefore, the farmer must make a planning without perfect information on yields.

Farmer example. Stochastic Model

The farmer is unable to make a perfect decision that would be the best in all circumstances.

DECISION PROCESS

Decisions on land assignments (i.e., x_1, x_2, x_3)

$\Downarrow$

weather conditions and yields (scenarios)

$\Downarrow$

sales and purchases depend on the yields

Farmer example. Stochastic Model

Let as consider $\Omega = \{1, 2, 3\}$ the set of scenarios (1=good season, 2= average season, 3= bad season).

Second stage variables have to be re-defined:

$$\left.\begin{array}{ll} z_i^\omega & \text{for } i = 1,2,3,4, \text{ and} \\ y_i^\omega & \text{for } i = 1,2, \end{array}\right. \quad \text{for } \omega \in \Omega$$

We assume:

1. the objective is to maximize the long-run profit (expected profit).
2. farmer is neutral about risk.
3. the scenarios have the same probability to appear $(1/3)$.

Farmer example. Stochastic Model

Objective function

$$
\begin{aligned}
\text{mín}\, z(x,y,z) = {} & 150x_1 + 230x_2 + 260x_3 \\
& - \frac{1}{3}(170z_1^1 - 238y_1^1 + 150z_2^1 - 210y_2^1 + 36z_3^1 + 10z_4^1) \\
& - \frac{1}{3}(170z_1^2 - 238y_1^2 + 150z_2^2 - 210y_2^2 + 36z_3^2 + 10z_4^2) \\
& - \frac{1}{3}(170z_1^3 - 238y_1^3 + 150z_2^3 - 210y_2^3 + 36z_3^3 + 10z_4^3),
\end{aligned}
$$

Farmer example. Stochastic Model

Constraints

$$
\begin{array}{lllllllllll}
x_1 + & x_2 + & x_3 & & & & & & & \leq 500 & \\
3x_1 & & & + y_1^1 - z_1^1 & & & & & & \geq 200 & \\
 & 3{,}6x_2 & & & + y_2^1 - z_2^1 & & & & & \geq 240 & \\
 & & -24x_3 & & & z_3^1 + z_4^1 & & & & \leq 0 & \\
 & & & & & z_3^1 & & & & \leq 6000 & \\
2{,}5x_1 & & & + y_1^2 - z_1^2 & & & & & & \geq 200 & \\
 & 3x_2 & & & + y_2^2 - z_2^2 & & & & & \geq 240 & \\
 & & -20x_3 & & & z_3^2 + z_4^2 & & & & \leq 0 & \\
 & & & & & z_3^2 & & & & \leq 6000 & \\
2x_1 & & & + y_1^3 - z_1^3 & & & & & & \geq 200 & \\
 & 2{,}4x_2 & & & + y_2^3 - z_2^3 & & & & & \geq 240 & \\
 & & -16x_3 & & & z_3^3 + z_4^3 & & & & \leq 0 & \\
 & & & & & z_3^3 & & & & \leq 6000 & \\
x_1 , & x_2 , & x_3 & & & & & & & \geq 0 & \\
 & & & y_1^\omega , z_1^\omega , & y_2^\omega , z_2^\omega , & z_3^\omega , z_4^\omega & & & & \geq 0 & \forall \omega \in \Omega
\end{array}
$$

Farmer example. Deterministic Equivalent Model

$$\begin{array}{lllll} \text{mín} & 150x_1 + 230x_2 + 260x_3 + E_\xi Q\big[\big(\mathbf{x}, \xi(\omega)\big)\big] \\ \text{s.a} & x_1 + \quad x_2 + \quad x_3 \leq 500 \\ & x_1 \;,\quad x_2 \;,\quad x_3 \geq 0 \end{array}$$

$$\begin{array}{rlllllll} Q\big(\mathbf{x}, \xi(\omega)\big) = \text{mín} & 238y_1 + 210y_2 - 170z_1 - 150z_2 - 36z_3 - 10z_4 \\ \text{s.a} & y_1 & - & z_1 & & & & \geq 200 - t_1^\omega x_1 \\ & & y_2 & & - & z_2 & & \geq 240 - t_2^\omega x_2 \\ & & & & & z_3 + & z_4 & \leq 0 - t_3^\omega x_3 \\ & & & & & z_3 & & \leq 6000 \\ & y_1 \;, & y_2 \;, & z_1 \;, & z_2 \;, & z_3 \;, & z_4 & \geq 0 \end{array}$$

where $\xi(\omega) = (t_1^\omega, t_2^\omega, t_3^\omega)$, $\xi(1) = (3; 3{,}6; 24)$, $\xi(2) = (2{,}5; 3; 20)$, $\xi(3) = (2; 2{,}4; 16)$.

Farmer example. Stochastic Model

Optimal solution to the stochastic model

	Above			Average			Below		
	Whea	Corr	Sugar Beets	Whea	Corr	Sugar Beets	Whea	Corr	Sugar Beets
Surface (Ha)	170	80	250	170	80	250	170	80	250
Yield (T)	510	288	6000	425	240	5000	340	192	4000
Sales (T)	310	48	6000	225	-	5000	140	-	4000
Purchase (T)	-	-	-	-	-	-	-	48	-
Utility	167000			109350			48820		
Overall profit	108390								

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Value of the information

In this case we can consider three situations:

- We know the future:
  - No uncertainty (perfect information).
  - The optimal solution for each scenario can be applied.
  - It is known as the wait and see (WS) solution.
- We have some information about the future:
  - Uncertainty included in the model (stochastic model).
  - Decisions take into account this information.
- We do not have any information about the future:
  - Uncertainty is not considered (deterministic model).
  - Decisions are independent of the scenarios.

Value of the information. Farmer example

Information about the uncertainty is included in the model

	Above			Average			Below		
	Wheat	Corn	Sugar Beets	Wheat	Corn	Sugar Beets	Wheat	Corn	Sugar Beets
Surface (Ha)	170	80	250	170	80	250	170	80	250
Yield (T)	510	288	6000	425	240	5000	340	192	4000
Sales (T)	310	48	6000	225	-	5000	140	-	4000
Purchase (T)	-	-	-	-	-	-	-	48	-
Utility	167000			109350			48820		
Overall profit	108390								

Perfect information

	Above (+20 %)			Average			Below (-20 %)		
	Wheat	Corn	Sugar Beets	Wheat	Corn	Sugar Beets	Wheat	Corn	Sugar Beets
Surface (Ha)	183,33	66,67	250	120	80	300	100	25	375
Yield (T)	550	240	6000	300	240	6000	200	60	6000
Sales (T)	350	-	6000	100	-	6000	-	-	6000
Purchase (T)	-	-	-	-	-	-	-	180	-
Overall profit	167667			118600			59950		

Value of the information. Farmer example

Perfect information

	Above (+20 %)			Average			Below (-20 %)		
	Wheat	Corn	Sugar Beets	Wheat	Corn	Sugar Beets	Wheat	Corn	Sugar Beets
Surface (Ha)	183,33	66,67	250	120	80	300	100	25	375
Yield (T)	550	240	6000	300	240	6000	200	60	6000
Sales (T)	350	-	6000	100	-	6000	-	-	6000
Purchase (T)	-	-	-	-	-	-	-	180	-
Overall profit	167667			118600			59950		

The mean profit over three years (and in the long run) is:

$$\frac{167667 + 118600 + 59950}{3} = 115406 \,€\,.$$

The stochastic model gives 108390 € of long run profit.

The difference between this values, 7016 € , represents what is called the *expected value of perfect information, EVPI*.

It represents the loss of profit due to the presence of uncertainty

Value of the information. Farmer example

In general:

- The optimum of scenario ω is $\min_{\mathbf{x}} z(\mathbf{x}, \xi^{\omega})$
- Wait-and-see (WS) solution:

$$WS = E_{\xi}\{\min_{\mathbf{x}} z(\mathbf{x}, \xi)\} = E_{\xi}\{z(\mathbf{x}(\xi), \xi)\}.$$

- Stochastic model solution:

$$RP = \min_{\mathbf{x}}\{E_{\xi} z(\mathbf{x}, \xi)\}.$$

- The *expected value of perfect information, EVPI*:

$$EVPI = RP - WS$$

.

Value of the information. Farmer example

Uncertainty is not considered in the model

The average scenario is used to obtain the optimal solution:

	Wheat	Corn	Sugar Beets
Surface (Ha)	120	80	300
Yield (T)	300	240	6000
Sales (T)	100	-	6000
Purchase (T)	-	-	-
Overall profit	118600		

The planting cost is:

$$150 * 120 + 230 * 80 + 260 * 300 = 114400 \text{ euros}$$

Value of the information. Farmer example

Uncertainty is not considered in the model

With this decision, if the favorable scenario happens, the best decision to be taken can be obtained by solving the following model:

$$
\begin{array}{llllllll}
114400 + \text{mín} & 238y_1 & -170z_1 & +210y_2 & -150z_2 & -36z_3 & -10z_4 & \\
\text{s.a.} & y_1- & z_1 & & & & & \geq 200 - 3 \cdot 120 \\
& & & y_2- & z_2 & & & \geq 240 - 3{,}6 \cdot 80 \\
& & & & & z_3+ & z_4 & \leq 24 \cdot 300 \\
& & & & & z_3 & & \leq 6000 \\
& y_1\ , & z_1\ , & y_2\ , & z_2\ , & z_3\ , & z_4 & \geq 0
\end{array}
$$

The optimal solution for the average scenario and the unfavorable can be also obtained.

Value of the information. Farmer example

Optimal solution for each scenario

	Above (+20 %)			Average			Below (-20 %)		
	Wheat	Corn	Sugar Beets	Wheat	Corn	Sugar Beets	Wheat	Corn	Sugar Beets
Surface (Ha)	120	80	300	120	80	300	120	80	300
Yield (T)	360	298	7200	300	240	6000	240	192	4800
Sales (T)	160	48	7200	100	-	6000	40	-	4800
Purchase (T)	-	-	-	-	-	-	-	48	-
Overall profit	148000			118600			55120		

The mean profit over three years (and in the long run) is:

$$\frac{1}{3}148000 + \frac{1}{3}118600 + \frac{1}{3}55120 = 107240\,€\,.$$

Value of the information. Farmer example

The average (deterministic) model gives 107240 € of long run profit.

The stochastic model gives 108390 € of long run profit.

The difference between these values, 1150 € , represents what is called the *value of the Stochastic Solution, VSS*.

Represents the lost of not considering the variability in the model

Value of the information. Farmer example

In general,

- The expected value (EV) problem is: $EV = \min_{\mathbf{x}} z(\mathbf{x}, \bar{\xi})$. Let us denote $\mathbf{x}(\bar{\xi})$ its optimal solution.
- The expected result of the expected value (EEV) is:

$$EEV = E_{\xi} z\big(\mathbf{x}(\bar{\xi}), \xi\big).$$

- The value of the stochastic solution (VSS) is

$$VSS = EEV - RP.$$

VSS assesses the value of knowing and using distributions on future outcomes.

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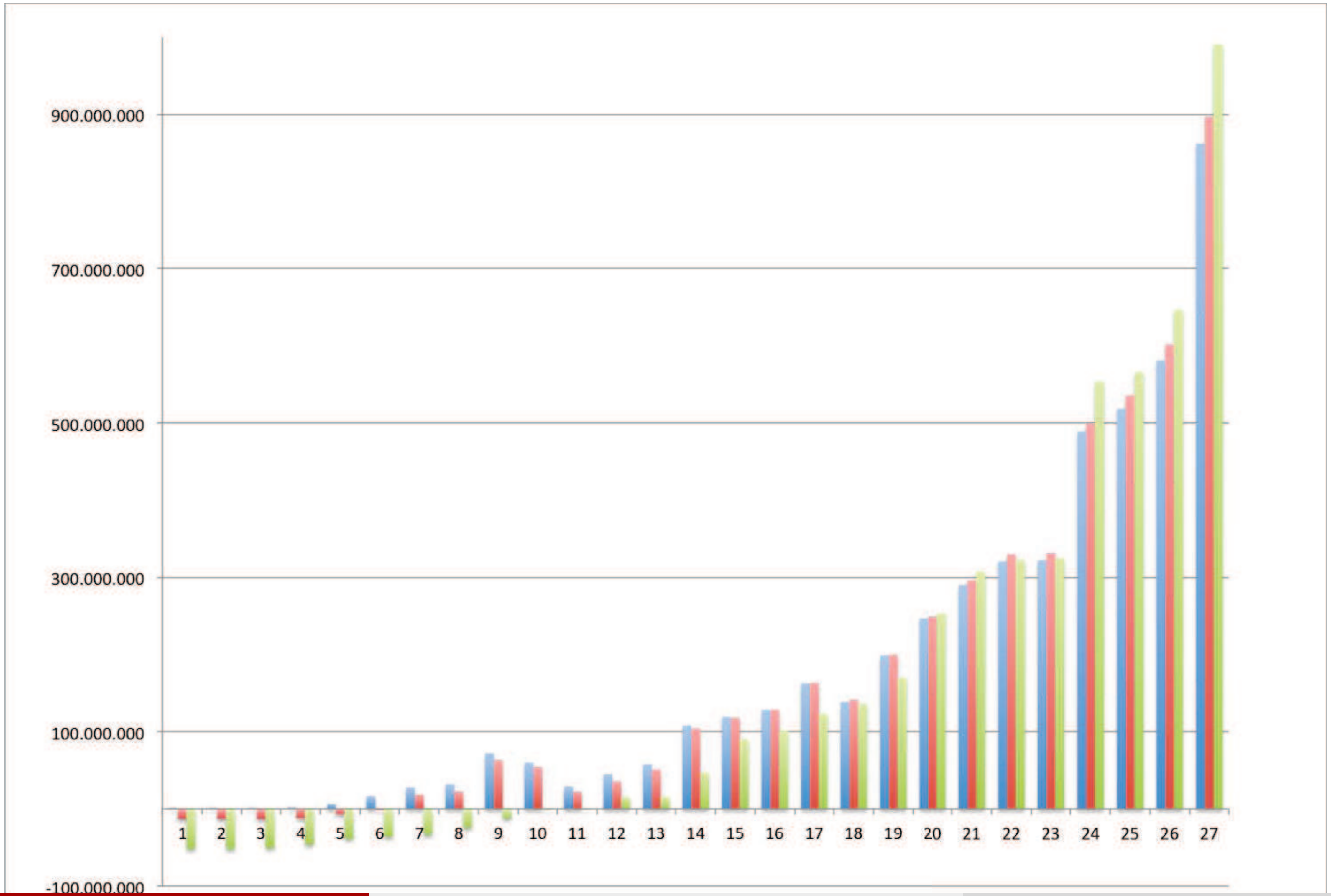
Introducción

- Stochastic Optimization allows to incorporate uncertainty into the model.
- It provides solutions for the first stage (implementable) valid for all scenarios and, in addition, taking into account that uncertainty.
- The classical approach optimizes the expected value:
  - It is only valid in long-term situations, where scenarios are repeated.
  - It doesn't take into account how bad a scenario can be.

Introducción

The figure below shows the profit associated with each of the 27 scenarios for three possible solutions to a mining problem:



Introduction

Average	337.8	345.3	354.1
% dif. from the best	4,62 %	2,50 %	
Máximum	862.0	896.7	991.0
Median	290.2	296.0	307.8
0,90-VaR	29.2	22.3	-11.8
Mínimum	1.5	-13.0	-52.3
Prob of losts	0.00	0.07	0.16
Average losts	-	-8.9	-25.8
0,90-CVaR	8.4	-5.7	-39.7

Green: risk neutral (maximizing the profit).

Blue and red consider risk aversion measures, penalizing bad solutions in the worst scenarios.

Risk management

Previous approach are based on the minimization of the expected cost.

Expected cost is a risk-neutral approach, when usually, decision-makers have some level of risk aversion.

Usually, risk is measure in terms of variability.

- Markowitz (1959) presents a model for minimizing the variance of a response variable by adding a constraint that guarantee a minimum level in the expected value.
- Charnes y Cooper (1959) present models for maximizing the probability of reaching a given aspiration level.

In most of the cases, these approaches are non-linear, and,therefore, cannot be applied in real-life problems.

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Markowitz's mean-risk model

A key point of Markowitz's work is that he proposes an optimisation model under uncertainty in terms of a balance between risk and profit.

If $R(\mathbf{x}, \xi)$ is a risk measure and $B(\mathbf{x}, \xi)$ a profit masure, the Markovitz model can be formulated as follows:

$$\text{mín}\, R(\mathbf{x}, \xi) \quad / \quad \mathbf{x} \in \mathcal{X}, \quad B(\mathbf{x}, \xi) \geq \pi_0$$

Alternatively,

- Maximization of profit, bounding the risk

$$\text{mín} -B(\mathbf{x}, \xi) \quad / \quad \mathbf{x} \in \mathcal{X}, \quad R(\mathbf{x}, \xi) \leq \rho_0$$

- Optimization of a weighted combination of risk and profit:

$$\text{mín} -B(\mathbf{x}, \xi) + \lambda R(\mathbf{x}, \xi) \quad / \quad \mathbf{x} \in \mathcal{X} \qquad\qquad (\lambda > 0)$$

Downside risk measures

Markowitz (1959) uses the semi-variance as the risk measure:

$$R(X) = \sigma_-^2(X) = E\left[(X - E(X))_-^2\right]$$

Other authors use the semi-deviation:

$$R(X) = E\left[(X - E(X))_-\right]$$

Semi-deviation can be generalized by considering the deviation from a certain level of aspiration set by the decision maker:

$$R(X) = E\left[(X - a)_-^p\right], \qquad p \geq 0,\ a \in \mathbb{R}$$

Another alternative is to measure the risk from the worst case scenario (*Worst Case Risk*) or (Robust Optimization).

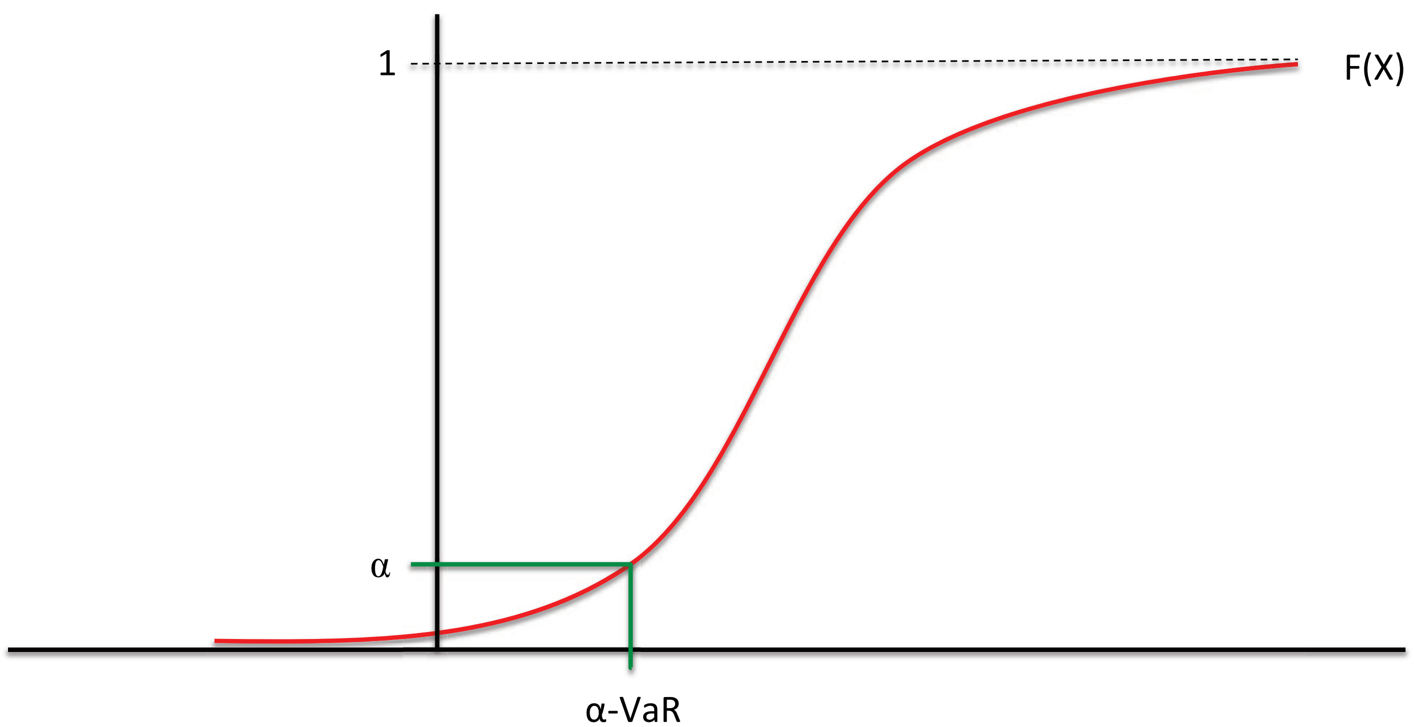
Value-at-Risk

Value-at-Risk (VaR) appears in the eities y se ha convertido en una de las medidas del riesgo más utilizadas (en finanzas).

The $VaR_\alpha(X)$ ($\alpha \in (0, 1)$) of a random variable X is the α-quantile of the distribution fuction:

$$VaR_\alpha(X) = P(X \leq \alpha)$$

It is an standard in finance:



Value-at-Risk

VaR has been widely criticized:

- It is not sub-additive.
- It does not take into account the tails of the distribution: VaR does not consider how bad the scenarios with a profit below VaR can be.
- It is neither convex nor continuous as α-function.

Then, new *coherent* risk measures has been proposed (Artzner et al., 1999):

A1 Monotonicity if $X \geq 0$, then $R(X) \leq 0$, for $X \in \mathcal{X}$.

A2 Sub-additivity: $R(X+Y) \leq R(X) + R(Y), \forall X, Y \in \mathcal{X}$.

A3 Positive homogeneity: $R(\lambda X) = \lambda R(X), \forall X \in \mathcal{X}$ and $\lambda > 0$.

A4 Translation invariance: $R(X+a) = R(X) - a, \forall X \in \mathcal{X}$ and $a \in \mathbb{R}$.

Conditional Value-at-Risk

The β-Conditional Value-at-Risk (β-CVaR), defined as conditional expectation of profit below α, takes into account the profit of these undesirable scenarios.

$$
\begin{array}{lll}
\text{máx} & \displaystyle\sum_{g\in\mathcal{G}} w_g(a_g x_g + c_g y_g) + & \\
& \displaystyle\rho\Big\{\alpha + \frac{1}{1-\beta}\sum_{d\in\mathcal{G}^T} w_d\Big(\sum_{g\in\mathcal{G}_d} w_g(a_g x_g + c_g y_g) - \alpha\Big)_+\Big\} & \\
\text{s.t.} & A' x_{\sigma(g)} + A x_g + B' y_{\sigma(g)} + B y_g = b & \forall g \in \mathcal{G} \\
& \displaystyle\sum_{g\in\mathcal{N}_d} (a_g x_g + c_g y_g) + M^\omega \nu^\omega \geq \alpha & \forall \omega \in \Omega \\
& \displaystyle\sum_{\omega\in\Omega} w^\omega \nu^\omega \leq 1 - \beta & \\
& x_g \in \{0,1\}^{n_t}, y_g \geq 0 & \forall g \in \mathcal{G} \\
& \nu^\omega \in \{0,1\} & \forall \omega \in \Omega.
\end{array} \tag{1}
$$

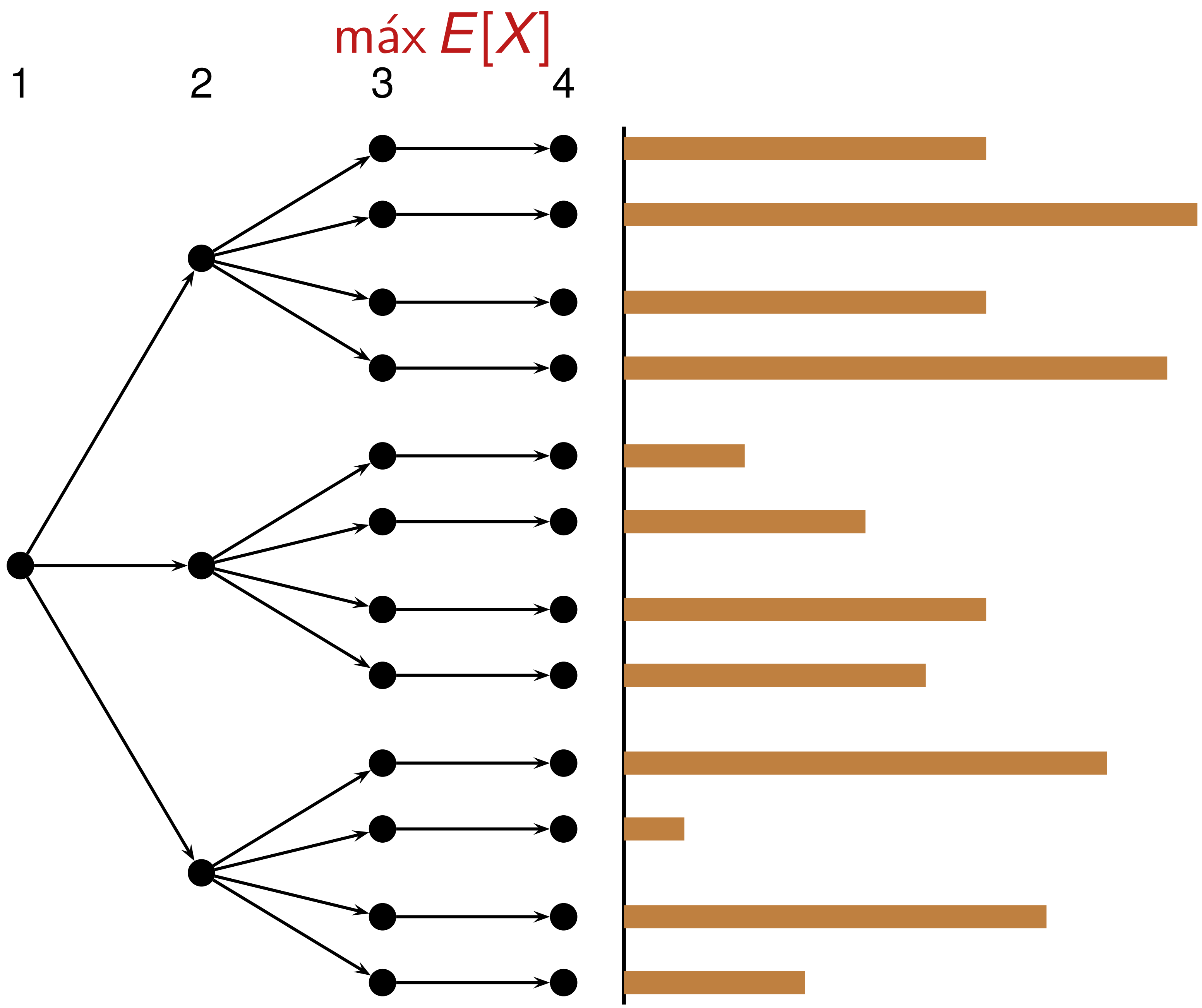
Deficit Probability (Schultz and Tiedemann 2003)

The objective is to minimize the probability of the scenarios where the profit is below a given threshold, say ϕ, provided by the modeler.

$$
\begin{aligned}
\text{máx} \textstyle\sum_{g\in\mathcal{G}} & w_g(a_g x_g + c_g y_g) - \ \rho \sum_{\omega\in\Omega} w^\omega \nu^\omega \\
\text{s.t.} & A' x_{\sigma(g)} + A x_g + B' y_{\sigma(g)} + B y_g = b && \forall g \in \mathcal{G} \\
& \sum_{g\in\mathcal{N}_d} (a_g x_g + c_g y_g) + M^\omega \nu^\omega \geq \phi && \forall \omega \in \Omega \\
& x_g \in \{0,1\}^{n_t}, y_g \geq 0 && \forall g \in \mathcal{G} \\
& \nu^\omega \in \{0,1\} && \forall \omega \in \Omega.
\end{aligned}
\tag{2}
$$

Risk management and stochastic optimization

Traditional approach: optimization of the expected profit



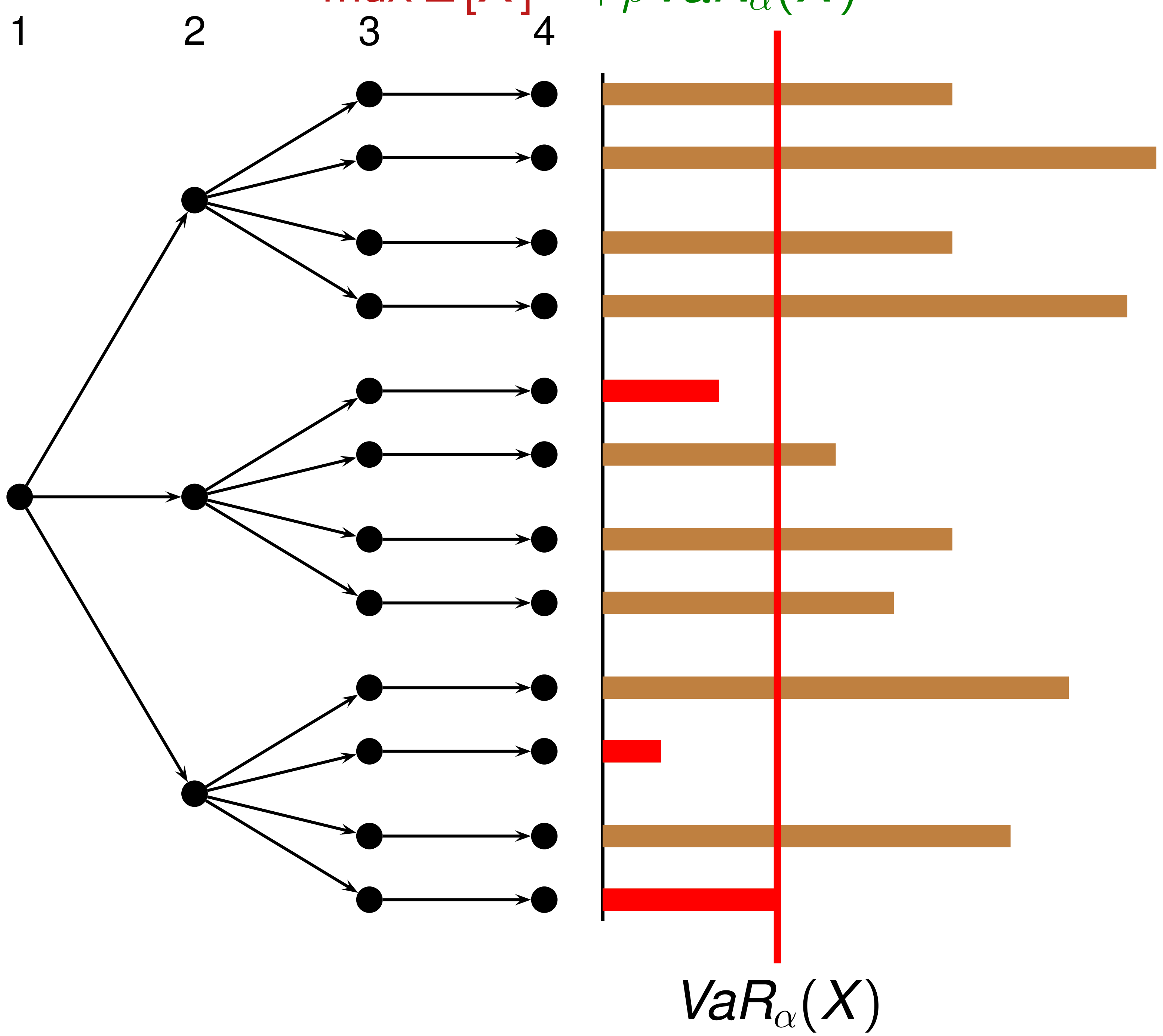
RN: Risk Neutral (Dantzig, 1955)

- Time consistent.
- No take into account the tails of the distribution.

Risk management and stochastic optimization

Traditional approach: optimization of the expected profit and risk control: VaR

$\max E[X] \quad +\rho VaR_{\alpha}(X)$



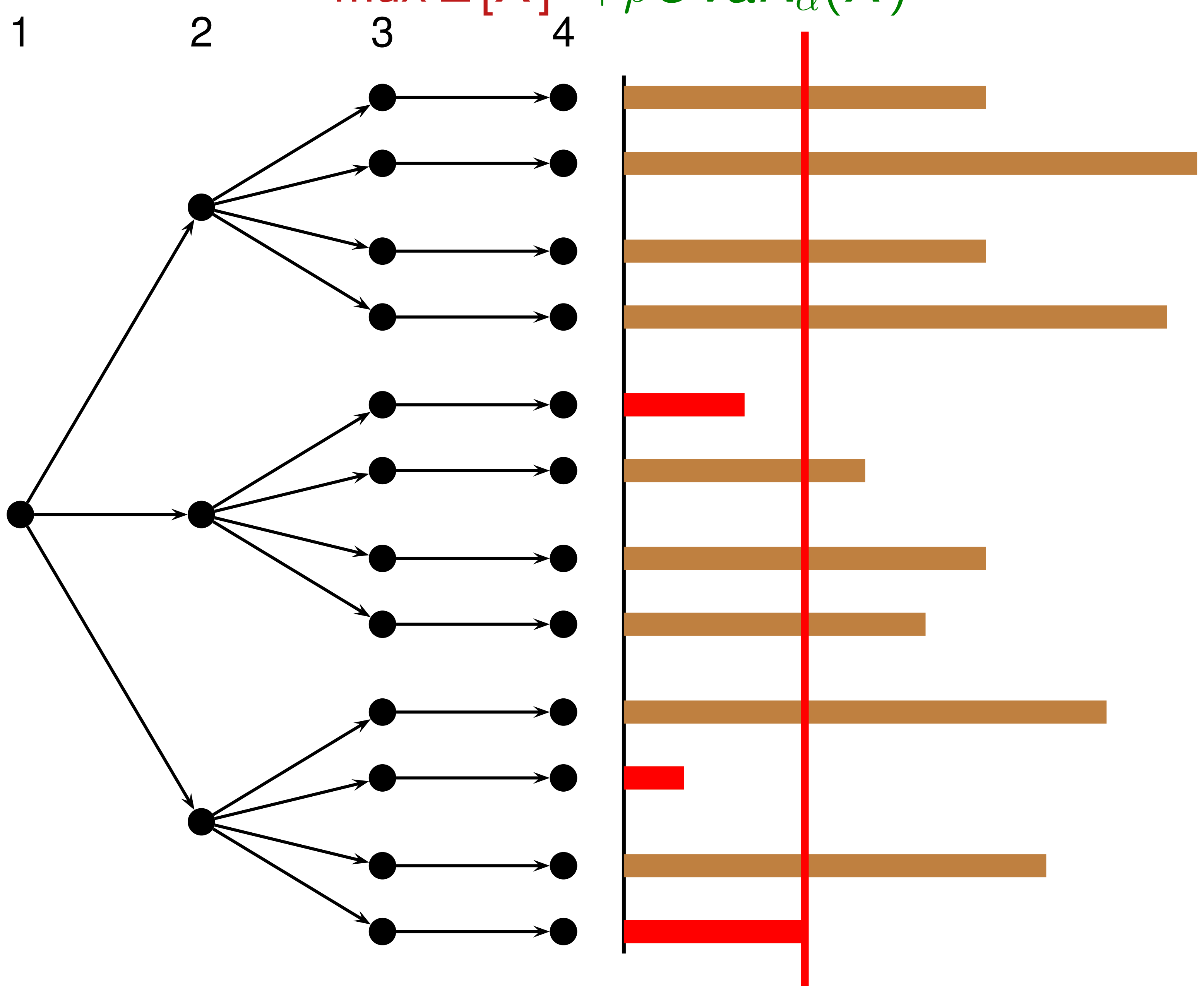
VaR: Value at Risk

- No coherent risk measure.
- It does no take into account the tails of the distribution.

Risk management and stochastic optimization

Traditional approach: optimization of the expected profit and risk control: CVaR

$\max E[X] + \rho CVaR_\alpha(X)$



$CVaR_\alpha(X)$
$E\big((X \,|\, X \leq VaR_\alpha(X)\big)$

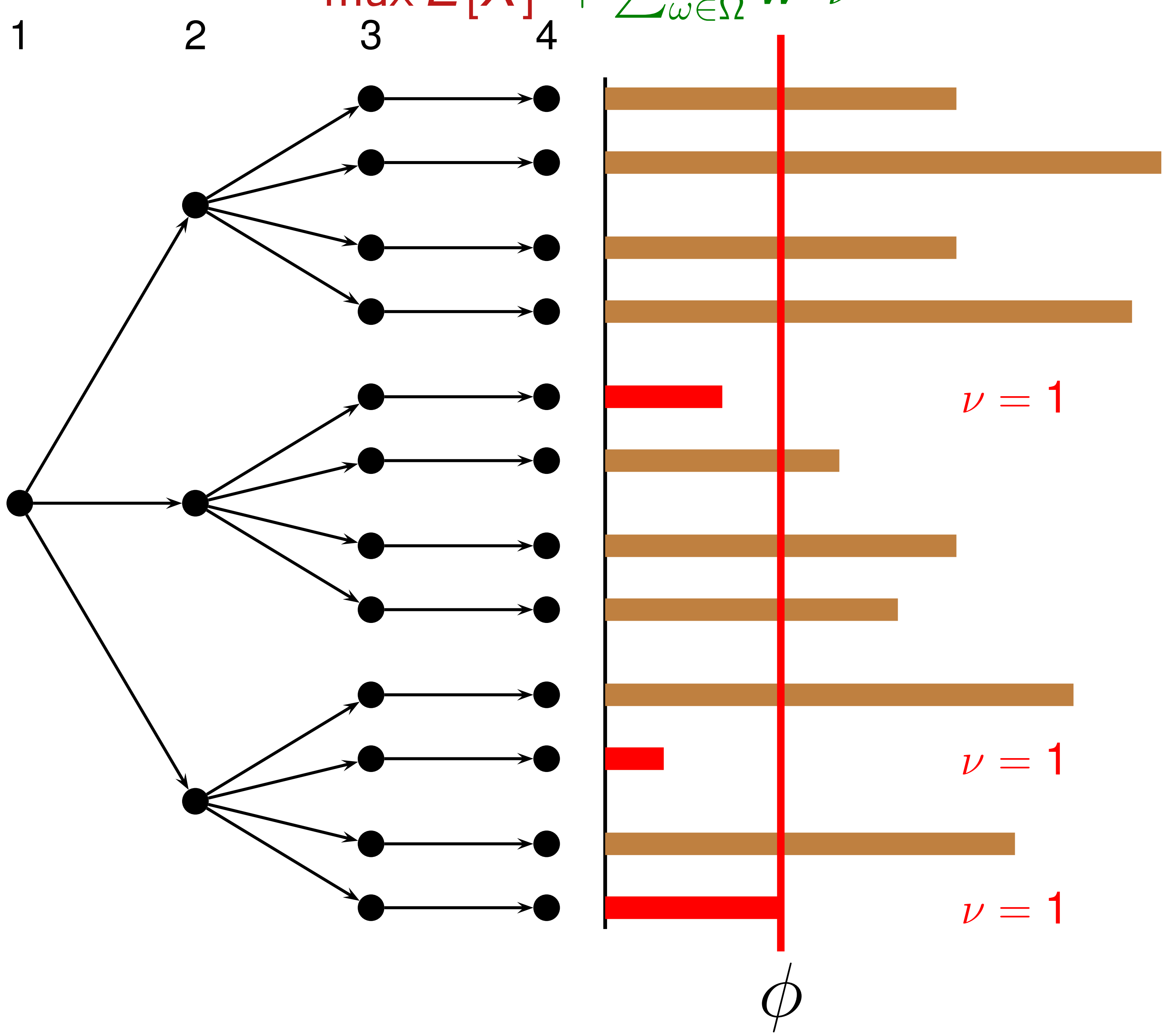
CVaR:
Conditional Value at Risk

- Coherent risk measure.
- No time consistent (the solution changes if the model is reoptimiced along the time horizon).
- It does no take into account risk at intermediate periods.

Risk management and stochastic optimization

Traditional approach: optimization of the expected profit and risk control: Deficit probability

$$\max E[X] + \sum_{\omega\in\Omega} w^{\omega}\nu^{\omega}$$



Deficit Probability

Contents

Stochastic dominance constraint strategies (sdc)

The expected objective function value is maximized, such that a set of thresholds on the objective function value for each scenario is intended to be satisfied, with either a bound on the probability of failure for each threshold (so called first-order sdc) or a bound on the expected deficit on reaching each threshold (so called second-order sdc).

Note: a threshold and the associated bound on the probability of failure or the associated expected deficit is called *profile*, and is provided by the modeler.

First-order stochastic dominance constraint strategy

The objective function is to maximize the net profit and different upper bounds β^p are added to the weight of scenarios with a profit below the thresholds ϕ^p, for $p \in \mathcal{P}$, where $\mathcal{P}$ is the set of profiles under consideration.

$$
\begin{array}{lll}
\text{máx} & \displaystyle\sum_{g\in\mathcal{G}} w_g(a_g x_g + c_g y_g) & \\
\text{s.t.} & A' x_{\sigma(g)} + A x_g + B' y_{\sigma(g)} + B y_g = b & \forall g \in \mathcal{G} \\
& \displaystyle\sum_{g\in\mathcal{N}_d} (a_g x_g + c_g y_g) + M^\omega \nu^{\omega p} \geq \phi^p & \forall \omega \in \Omega, p \in \mathcal{P} \\
& \displaystyle\sum_{\omega\in\Omega} w^\omega \nu^{\omega p} \leq 1 - \beta^p & \forall p \in \mathcal{P} \\
& x_g \in \{0,1\}^{n_t}, y_g \geq 0 & \forall g \in \mathcal{G} \\
& \nu^{\omega p} \in \{0,1\} & \forall \omega \in \Omega, p \in \mathcal{P},
\end{array}
$$

where $\nu^{\omega p}$ is a 0-1 variable with value 1 if the objective function value for scenario ω is smaller than the threshold ϕ^p and 0 otherwise.

Second-order stochastic dominance constraint strategy

The second-order stochastic dominance constraint strategy requires a set of profiles given by the pairs $(\phi^p, e^p)\,\forall p \in \mathcal{P}$, where e^p is the upper bound of the expected deficit of the profit over the scenarios on reaching the threshold ϕ^p.

$$
\begin{array}{lll}
\text{máx} & \sum_{g\in\mathcal{G}} w_g(a_g x_g + c_g y_g) & \\
\text{s.t.} & A' x_{\sigma(g)} + A x_g + B' y_{\sigma(g)} + B y_g = b & \forall g \in \mathcal{G} \\
 & \phi^p - \sum_{g\in\mathcal{N}_d} (a_g x_g + c_g y_g) \le v^{\omega p} & \forall \omega \in \Omega, p \in \mathcal{P} \\
 & \sum_{\omega\in\Omega} w^{\omega} v^{\omega p} \le e^p & \forall p \in \mathcal{P} \\
 & x_g \in \{0,1\}^{n_t}, y_g \ge 0 & \forall g \in \mathcal{G} \\
 & v^{\omega p} \ge 0 & \forall \omega \in \Omega, p \in \mathcal{P},
\end{array}
$$

where $v^{\omega p}$ is a non-negative variable equal to the difference (if it is positive) between the threshold ϕ^p and the profit for scenario ω.

Contents

Probabilistic Programming (Charnes y Cooper, 1959)

In probabilistic programming some of the constraints or the objective are expressed in terms of probabilistic statements about first-stage decisions:

$$P_\xi\big(\mathbf{A}_i(\xi)\mathbf{x} \geq \mathbf{h}_i(\xi)\big) \geq \alpha_i, \qquad i = 1, 2, \ldots,$$

where $0 \leq \alpha_i < 1$ is some confidence level.

This is particularly useful when the cost and benefits of second stage decisions are difficult to asses.

Probabilistic Programming. Example

Let us consider the following covering location problem.

We have a set of n potential locations, with a investment cost c_j and we have to cover a set of m customers.

$$\min \sum_{j=1}^{n} c_j x_j$$

$$\text{s.a.} \sum_{j \in N_i} x_j \geq 1 \qquad \forall i = 1, \ldots, m$$

$$x_j \in \{0, 1\} \qquad \forall j = 1, \ldots, n$$

This model assumes that the location is always available.

Real-life problem usually considers that, when a service is required, the location cannot be available with a certain probability.

Probabilistic Programming. Example

Stochastic model:

> First stage: location decision.
>
> Second stage: decision about which clients are served.

In the stochastic model, deterministic covering constraints are substituted by the following probabilistic conditions:

$$P(\text{there is at least one location available to serve client } i) \geq \alpha,$$

where α is a confidence level, usually 90 % or 95 %.

Probabilistic Programming. Example

Let us suppose that a location is not available with probability q.

The probability of not servicing client j is $q^{\sum_{j\in N_i} x_j}$

The probabilistic condition is:

$$1 - q^{\sum_{j\in N_i} x_j} \geq \alpha \qquad \text{or} \qquad \sum_{j\in N_i} x_j \geq \left\lceil \frac{\ln(1-\alpha)}{\ln q} \right\rceil \qquad \forall i = 1, \ldots, m,$$

where $\lceil a \rceil$ denotes the smallest integer greater or equal to a.

For $q = 0{,}2$ and $\alpha = 95\,\%$, the deterministic equivalent model is:

$$\text{mín} \sum_{j=1}^{n} c_j x_j$$

$$\text{sunject to} \sum_{j\in N_i} x_j \geq 2 \qquad \forall i = 1, \ldots, m$$

Brief overview of risk averse measures

- Scenario immunization: Dembo ANOR'91; LFE Unicom'95
- Semi-deviations: Ogryczak & Ruszczynski EJOR'99; Ahmed MP'06.
- Expected VaR: Gaivoronski & Pflug in WS'99, JoR'05; Pflug inBook'00; Charpentier & Oulidi MMOR'09;
- Conditional VaR (CVaR): Rockafellar & Uryasev JoR'00; Ahmed MP'06; Schultz & Tiedemann MP'06; Fabian et al. SIOPT'15; Y. Huang & Guo SIOPT'16.
- Excess probabilities: Schultz & Tiedemann SIOPT'03.
- First- and second-order stochastic dominance (SD) constraints recourse-integer: Fabian EJOR'08; Gollmer, Neise & Schultz SIOPT'08; Gollmer, Gotzes & Schultz MP'11; LFE, Garín, Merino & Pérez EJOR'16; LFE, Garín & Unzueta COR'17.

Brief overview of risk averse measures

- Expected CVaR: Shapiro ORL'09; Ruszczynski MP'10; Rudloff, Street & Valladao EJOR'14; Asamov & Ruszczynski MPA'15; Homem-de-Mello & Pagnoncelli EJOR'16 ; Pflug & Pichler MOR'16; A-A, LFE, Guignard & Weintraub, submitted 2nd revision, 2017.
- Expected Conditional SD: LFE, Monge & RomeroMorales, submitted 2nd revision, 2017; LFE, Garín, Monge & Unzueta (to be submitted).

Time consistency property

One desirable property for a multiperiod model solution is time consistency.

- Let us assume that the decisions in a given problem up to any node g of the scenario tree have been made, for $g \in \mathcal{G}$, according to the solution obtained in the original model 'solved' at period $t = 1$.
- Then, the rationale behind a time-consistent risk averse measure is that the solution value to be obtained for the nodes in its successor set in the scenario tree for the related submodel 'solved' at stage $t(g)$ should have the same value as in the original model 'solved' at period t=1.
- The family of *expected conditional risk averse measures* (ECVaR) considered in Homem-de-Mello & Pagnoncelli EJOR'16, among other are the time-consistency.
- Notice that the proof only requires that the measure has the properties of translation-invariance and monotonicity.

  See some variants in Pflug & Pichler MOR'16, EJOR'16; Rudloff, Street & Valladao EJOR'14; Ruszczynski MP'10; Shapiro ORL'09, among others.

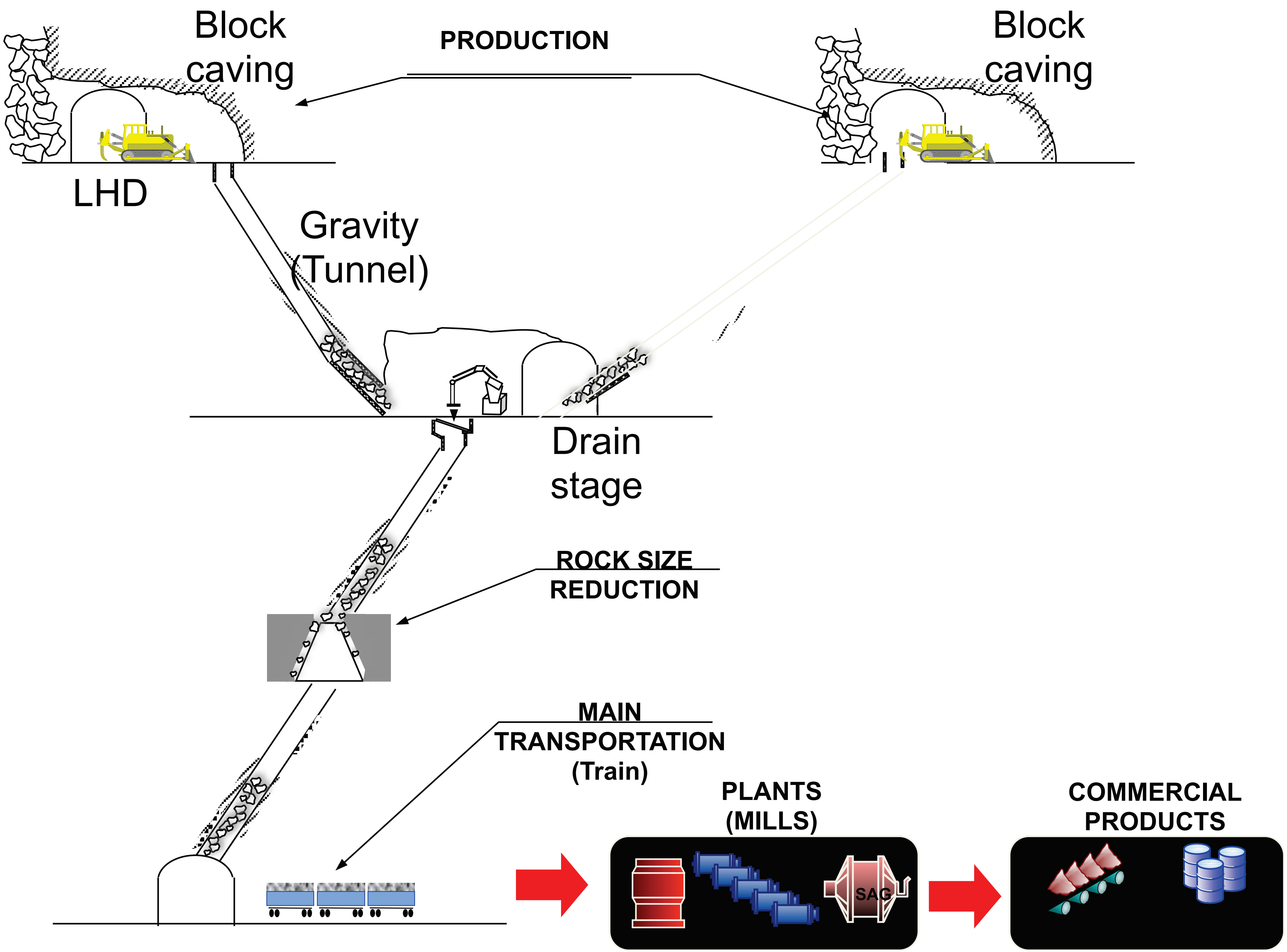
Contents

Description

- Mining is carried out in several sectors of the mine El Teniente (Chile).
- The mine is divided in 18 sectors, but only 3 are considered as active in the horizon (five years).
- The extraction can only be carried out either on the two smaller sectors, or on the largest one.
- There is a cost associated with increasing or decreasing production in a sector from one period to the next one.

Description

- In each sector there are a number of vertical columns, composed of blocks of 30 meters by 30 meters by 30 meters.
- The columns are adjacent to each other, and are extracted in sequence.
- A column has a height between 549m and 959m. Thus, a column may consist of between 18 and 32 vertical blocks.
- The extraction method used is called block caving: At each drawpoint of a column, a void is created so that the rock breaks and falls due to gravity.
- The following specific rules must be respected in the mining process:
  - The columns enter production in a specified sequence,
  - The extraction of columns must have properties of neighborhood smoothness, implying that after extraction, adjacent columns cannot have much difference in height in what remains, and
  - At each drawpoint there is a maximum extraction rate to prevent the roof from collapsing, as well as a minimum number of blocks to be extracted from each column to ensure a proper structure of the remaining mine.


Block caving
PRODUCTION
Block caving
LHD
Gravity (Tunnel)
Drain stage
ROCK SIZE REDUCTION
MAIN TRANSPORTATION (Train)
PLANTS (MILLS)
SAG
COMMERCIAL PRODUCTS

Description

- Decisions:
    - When to cave in each column,
    - when to move to the next column, and
    - how far up in the column to extract.
- The rate of copper content tends to drop as we go up the column.
- Depending on copper prices, it may be preferable at some point to drop the present column and move to the next one.
- Once a column is dropped from production, it cannot be re-entered due to mechanical and stability issues.

Description

For reducing the size of the deterministic version of the problem an aggregation procedure based on a cluster analysis is used (Weintraub et al, 2007).

The blocks of the original problem were aggregated based on spatial neighborhoods and similarities on the grade contents in copper and molibdenum, i.e., tons produced and extraction speed.

50 Blocks

Block

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Aggregation process

Cluster

1	2	3	4	4	4	5	5	5	5
6	7	8	8	9	9	9	9	10	10
11	12	13	13	14	14	14	15	10	16
11	12	12	17	17	17	15	15	15	16
18	18	19	19	20	20	20	20	16	16

Contents

Sets

- $\mathcal{T}$, set of periods in the time horizon.
- $\mathcal{S}$, set of sectors, where $\mathcal{S} = \{ES, FW, NN\}$ and ES, FW, NN are the three given sectors.
- $\mathcal{K}^s$, set of clusters in sector s, for $s \in \mathcal{S}$.
- $\mathcal{T}^k$, set of periods when cluster k can ben reached in the extraction process, for $k \in \mathcal{K}^s, s \in \mathcal{S}$.
- $\mathcal{P}^s$, set of subsets of clusters in sector s, such that each element $\mathcal{P}_i$ in $\mathcal{P}^s$ is a set of clusters that must be extracted simultaneously, for $s \in \mathcal{S}$.
- $Pred_k$, set of predecessor clusters of cluster k, such that all clusters in $Pred_k$ must be extracted by the time cluster k is extracted, for $k \in \mathcal{K}^s, s \in \mathcal{S}$.

Variables

0-1 variables

- z_{kt} takes the value 1 if cluster k is extracted in period t and 0 otherwise, for $t \in \mathcal{T}^k, k \in \mathcal{K}^s, s \in \mathcal{S}$.
- x_s takes the value 1 if sector s is extracted and 0 otherwise, for $s \in \mathcal{S}$.

Continuous variables

- ton_{st}, number of tons of rock extracted in sector s at period t, for $s \in \mathcal{S}, t \in \mathcal{T}$.
- ton^+_{st} and ton^-_{st}, increase and decrease in the number of tons extracted in sector s at period t, for $s \in \mathcal{S}, t \in \mathcal{T}$.
- ton^{B}_t and ton^{C}_t, number of tons sent to process in period t in processing streams B and C, respectively, for $t \in \mathcal{T}$.

Objective function

The objective is to maximize the net present value of the total profit along the time horizon:

- The income from selling the extracted copper and molibdenum,
- Reduced by:
  - the mining and sector costs,
  - the cost of production increase and decrease from one period to the next one,
  - and processing costs.

Constraints

- Sector selection: if sector *EN* is selected, none of the sectors *FW* and *NN* are selected, and vice-versa.

$$x_{ES} + x_{FW} \leq 1$$
$$x_{ES} + x_{NN} \leq 1$$

- No cluster is processed in an unselected sector:

$$\sum_{t \in \mathcal{T}^k} z_{kt} \leq x_s, \qquad \forall k \in \mathcal{K}^s, s \in \mathcal{S}$$

- If a cluster is processed at a given period then all predecessor clusters are also processed *by* that period:

$$\sum_{t' \in \mathcal{T}^k : t' \leq t} z_{kt'} \leq \sum_{t' \in \mathcal{T}^k : t' \leq t} z_{jt'} \qquad \forall j \in Pred_k, t \in \mathcal{T}^k, k \in \mathcal{K}^s, s \in \mathcal{S}$$

Constraints

- Clusters in set $\mathcal{P}_i$ have to be extracted simultaneously:

$$z_{kt} = z_{k't} \qquad \forall t \in \mathcal{T}^k, k, k' \in \mathcal{P}_i, \mathcal{P}_i \in \mathcal{P}^s, s \in \mathcal{S}$$

- Evaluation the number of tons processed:

$$ton_{st} = \sum_{k \in \mathcal{K}^s : t \in \mathcal{T}^k} TON_k \, z_{kt} \qquad \forall s \in \mathcal{S}, t \in \mathcal{T}$$

- Flow conservation constraints for the processing stream.

$$ton_t^{\mathrm{B}} + ton_t^{\mathrm{C}} = \sum_{s \in \mathcal{S}} ton_{st} \qquad \forall t \in \mathcal{T}$$

- Increase and decrease in the number of tons processed:

$$ton_{st}^{+} - ton_{st}^{-} = \begin{cases} ton_{st} - TON_s^{\mathrm{ini}}, & \text{if } t = 1 \\ ton_{st} - ton_{s,t-1}, & \text{if } t > 1 \end{cases} \qquad \forall s \in \mathcal{S}, t \in \mathcal{T}$$

Constraints

- Upper and lower bounds for the total area processed in each sector.

$$\underline{area}_s x_s \leq \sum_{k \in \mathcal{K}^s} area_k \sum_{t \in \mathcal{T}^k} z_{kt} \leq \overline{area}_s x_s \qquad \forall s \in \mathcal{S}$$

- Bounds on the number of tons processed in each period.

$$\sum_{s \in \mathcal{S}} ton_{st} \leq \overline{TON} \quad \forall t \in \mathcal{T}$$

$$\underline{TON}_{st} x_s \leq ton_{st} \quad \forall s \in \mathcal{S}, t \in \mathcal{T}$$

- Upper bound due to the capacity of processing stream B.

$$0 \leq ton_t^{\mathrm{B}} \leq \overline{TON}_t^{\mathrm{B}} \qquad \forall t \in \mathcal{T}$$

- Bounds on the maximum increase and decrease of tons in each sector in each period.

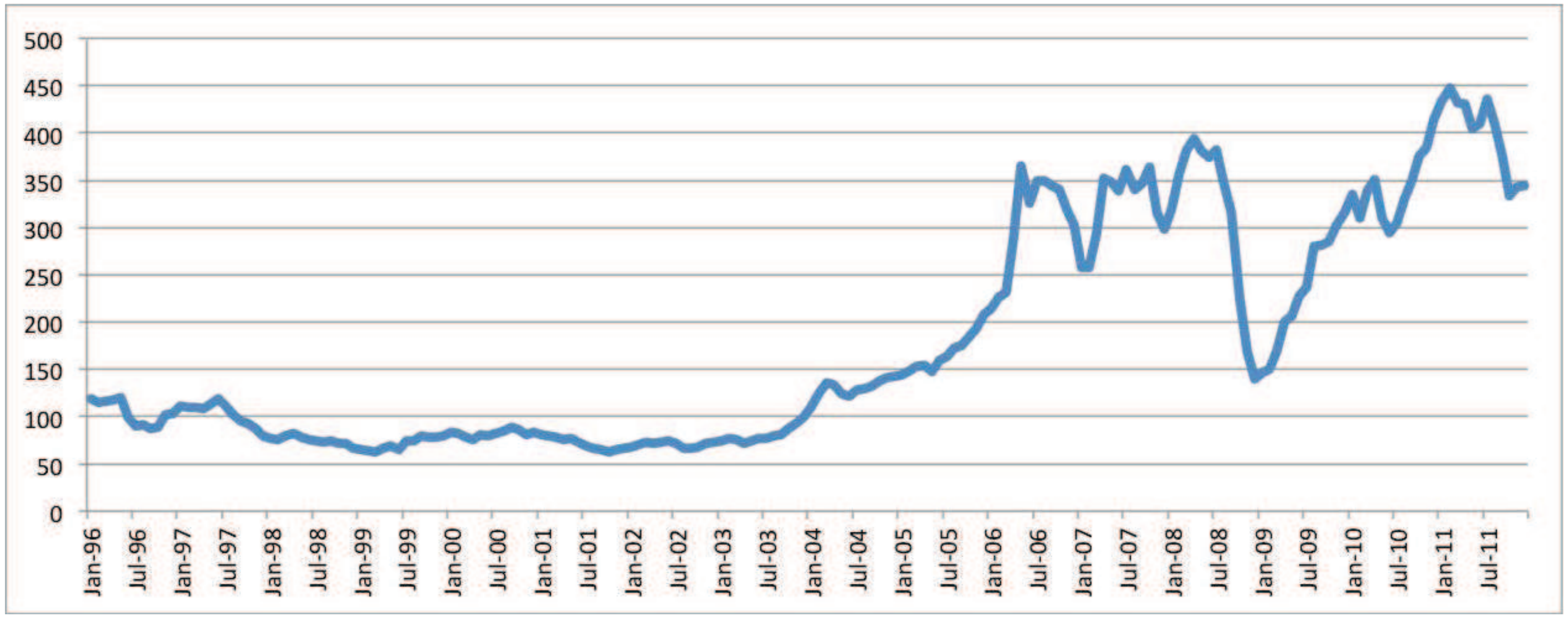
$$0 \leq ton_{st}^{+} \leq \overline{TON}_{st}^{+} x_s \quad \forall s \in \mathcal{S}, t \in \mathcal{T}$$

$$0 \leq ton_{st}^{-} \leq \overline{TON}_{st}^{-} x_s \quad \forall s \in \mathcal{S}, t \in \mathcal{T} - \{1\}$$

Uncertainty in the (future) prices of copper

The deterministic model assumes that prices are known in advance of the planning decision.

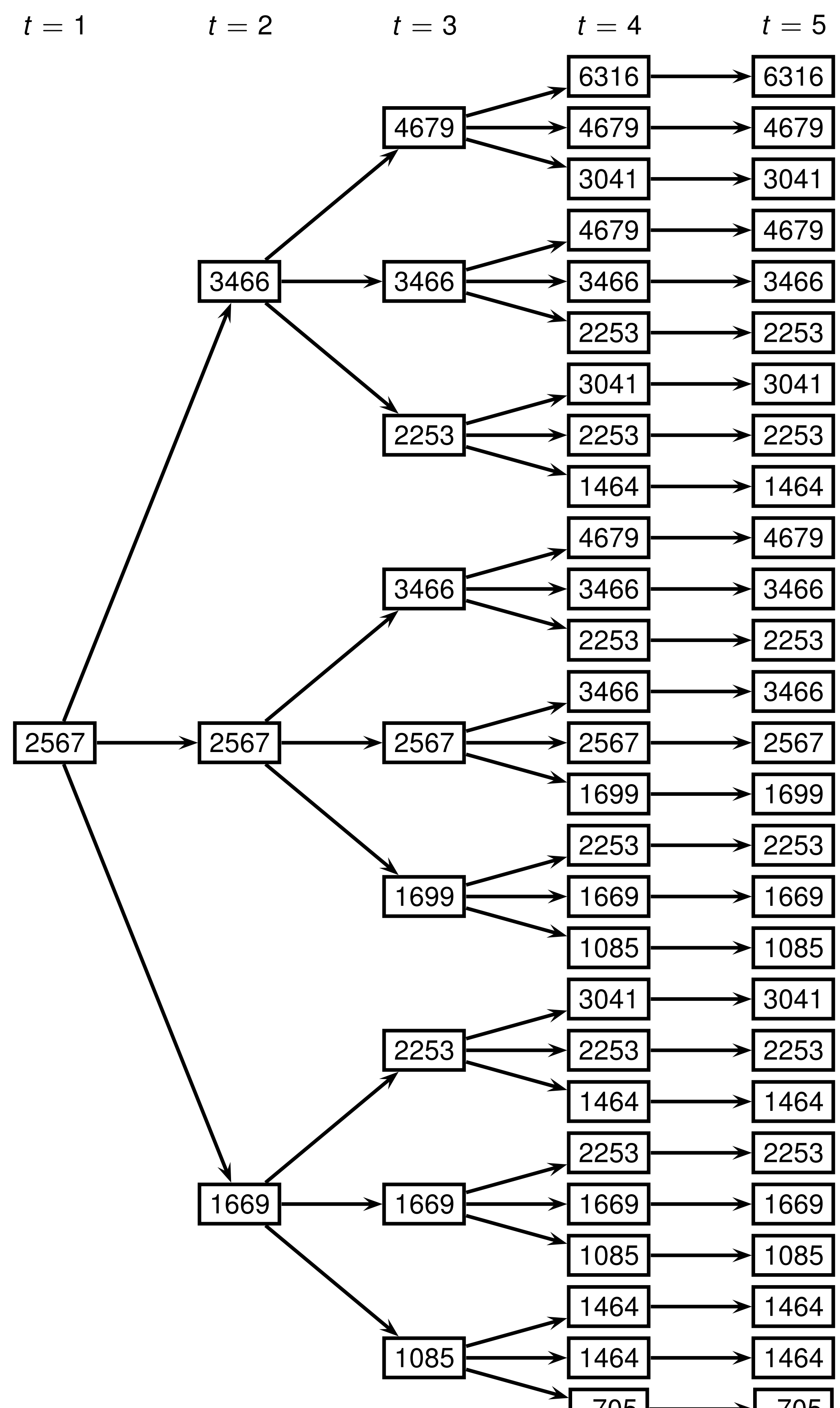
However, copper prices can vary along the planning horizon.



Historical data for copper prices (USD cents/LB)

Notice the volatility of the uncertain parameters which are therefore very difficult to predict.

A multistage scenario tree



#sce	ω
1	0,1250
2	0,0833
3	0,0417
4	0,0833
5	0,0556
6	0,0278
7	0,0417
8	0,0278
9	0,0139
10	0,0833
11	0,0556
12	0,0278
13	0,0556
14	0,0370
15	0,0185
16	0,0278
17	0,0185
18	0,0093
19	0,0417
20	0,0278
21	0,0139
22	0,0278
23	0,0185
24	0,0093
25	0,0139
26	0,0093
27	0,0046

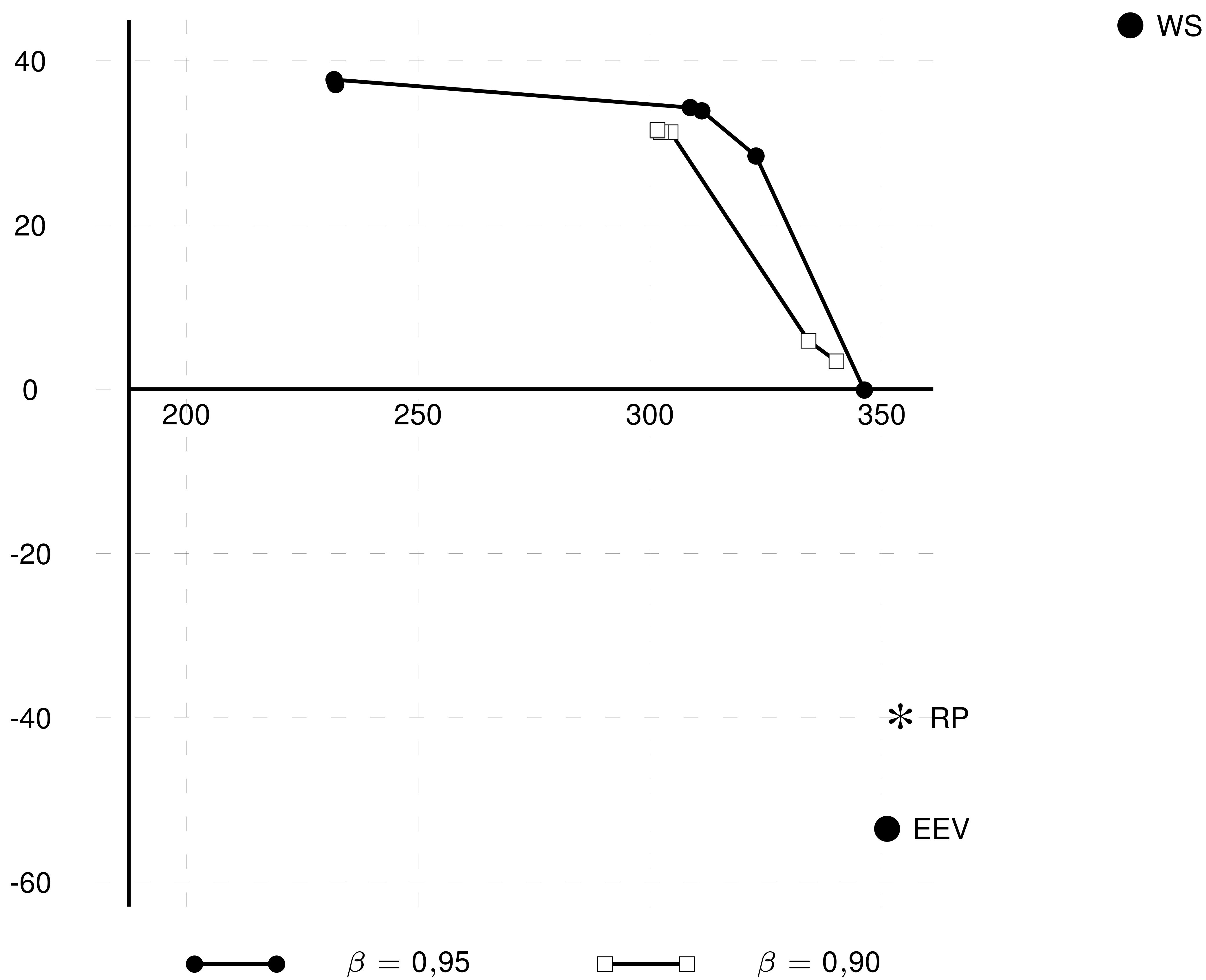
Model dimensions

	m	$n01$	nc	den
Deterministic	10708	6613	55	0.067
27-scen tree risk neutral DEM	288272	100974	737	0.004
45-scen tree risk neutral DEM	480490	167951	823	$<$ 0.004
75-scen tree risk neutral DEM	800694	274813	1992	$<$ 0.002

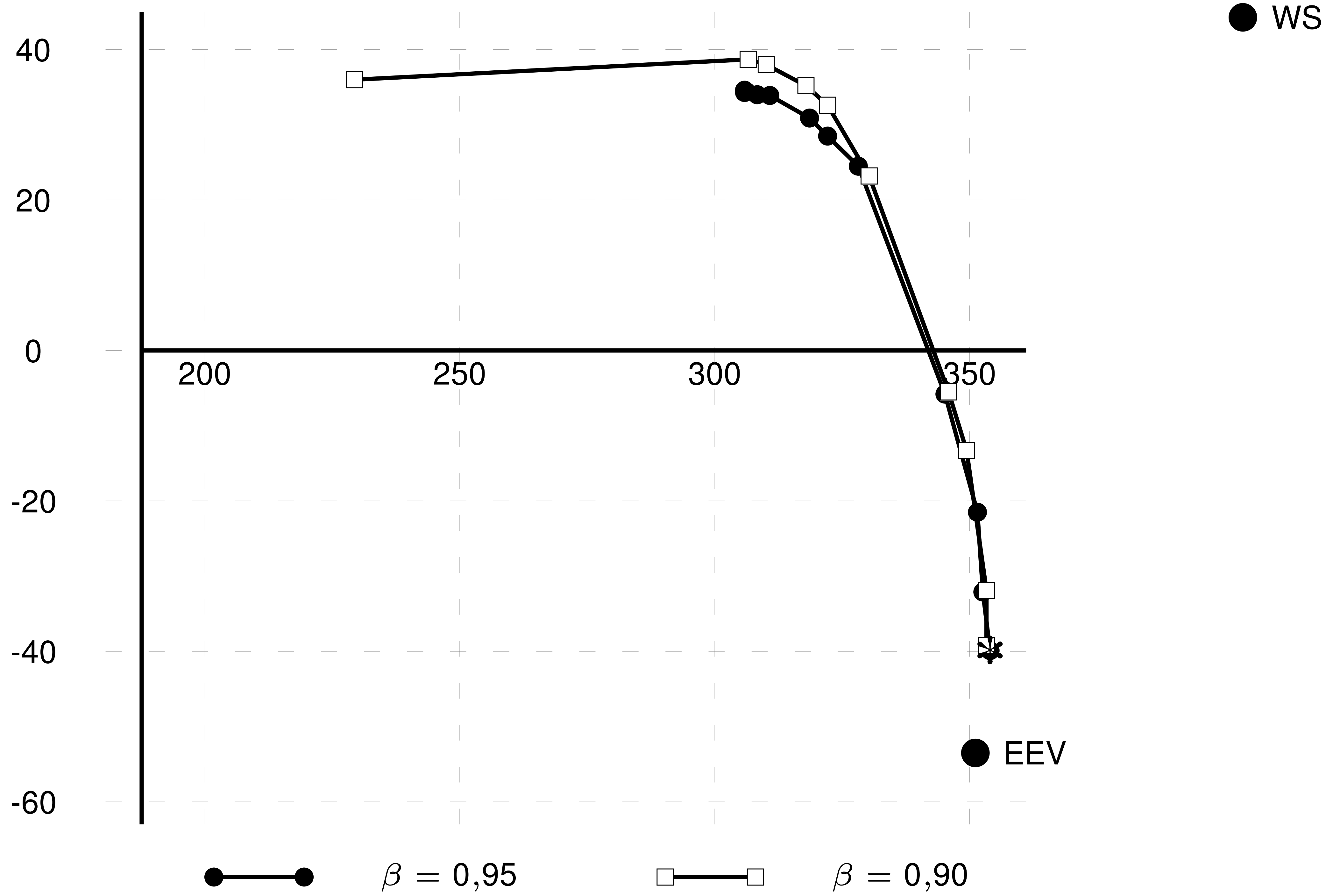
27-scen tree. Risk neutral versus deterministic solutions. WS , RP and EEV

	WS	RP	EEV
Solution value	404,19	354.18	352,18
Optimality GAP (%)	0,02	0,15	0,02
Greatest scen solution value	1031,72	991.02	986,39
Median	314,72	307.81	310,38
0,90-VaR	71,74	-11.85	-19,20
0,95-VaR	39,73	-39.10	-51,74
0,90-CVaR	44,33	-39.77	-53,53
Smallest scen solution value	28,88	-52.34	-66,67
Weight of scenarios with negative profit	0,000	0,157	0,157
Conditional expected negative profit	0,00	-25.89	-37,41
CPU time	207	241	80

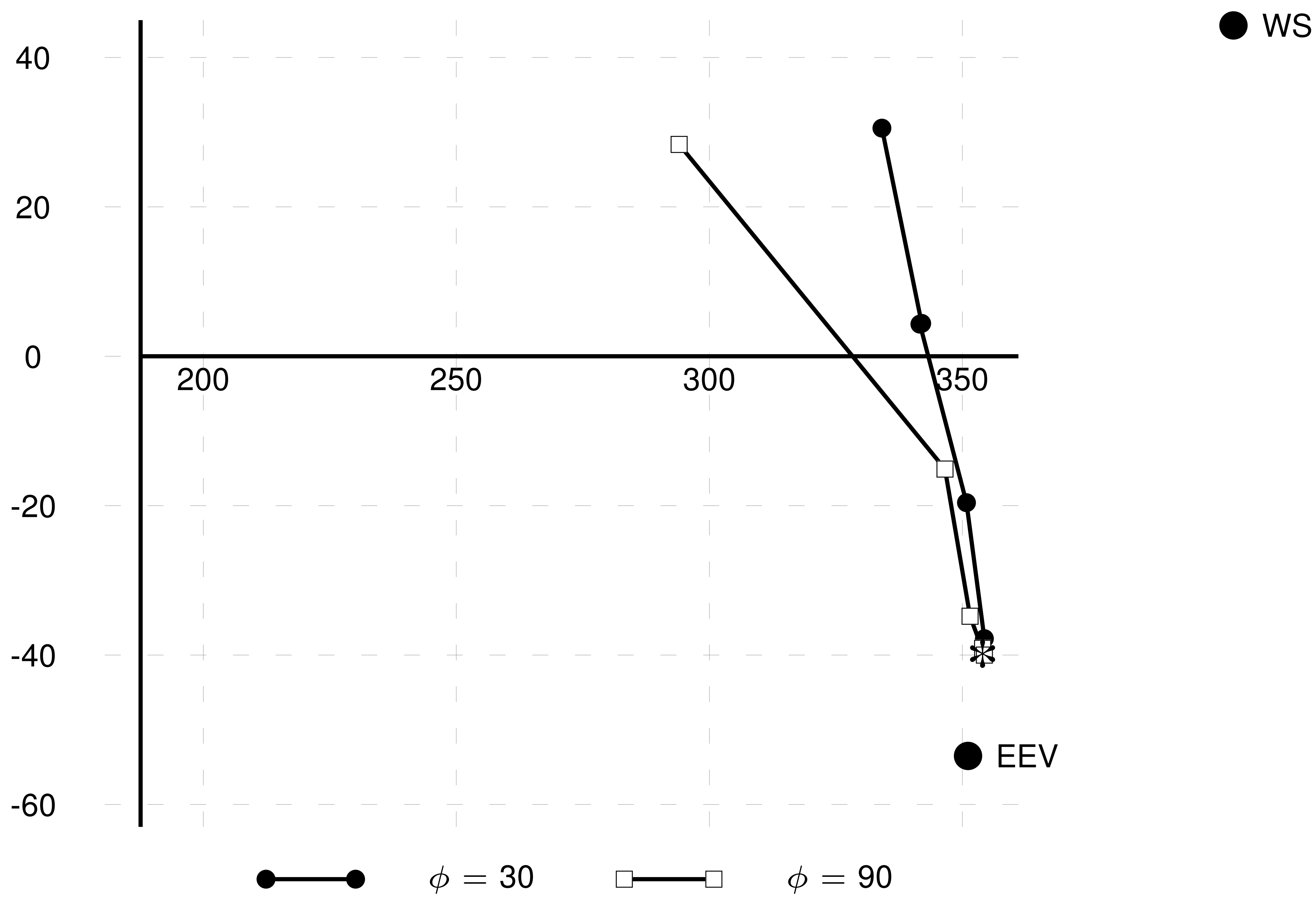
Q_e & VaR



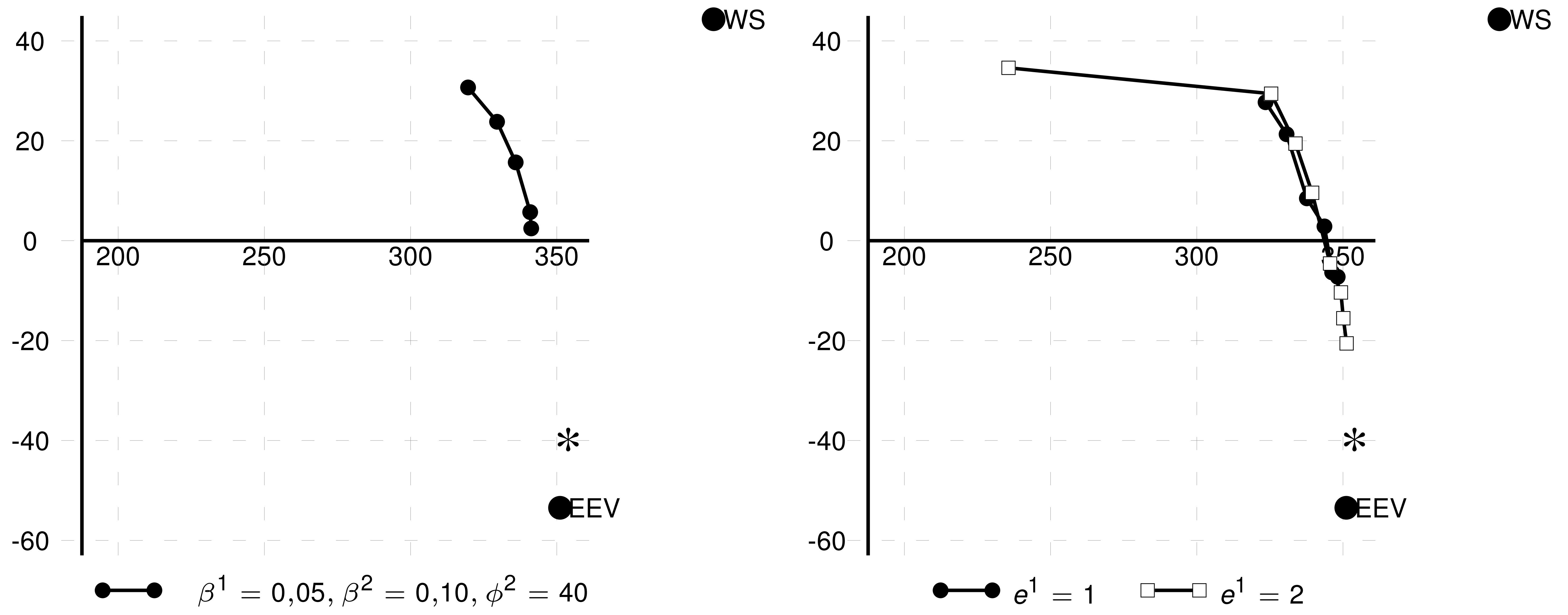
Q_e & CVaR



Q_e & DP



Stochastic dominance



Conclusions

- We have presented the stochastic version of the copper extraction planning problem along a time horizon (i.e, years) under uncertainty in the (volatile) copper prices.
- Even the risk neutral approach, provides a better solution than the traditional (and myopic) deterministic solution by considering the expected value of the uncertain parameters
- Risk adverse Expected value & CVaR strategy and the second-order stochastic dominance constraints (sdc) strategy seem to provide better results in the solution's quality (since they reduce the risk of bad scenarios without reducing too much the expected profit) and they require less elapsed time than any other ones.

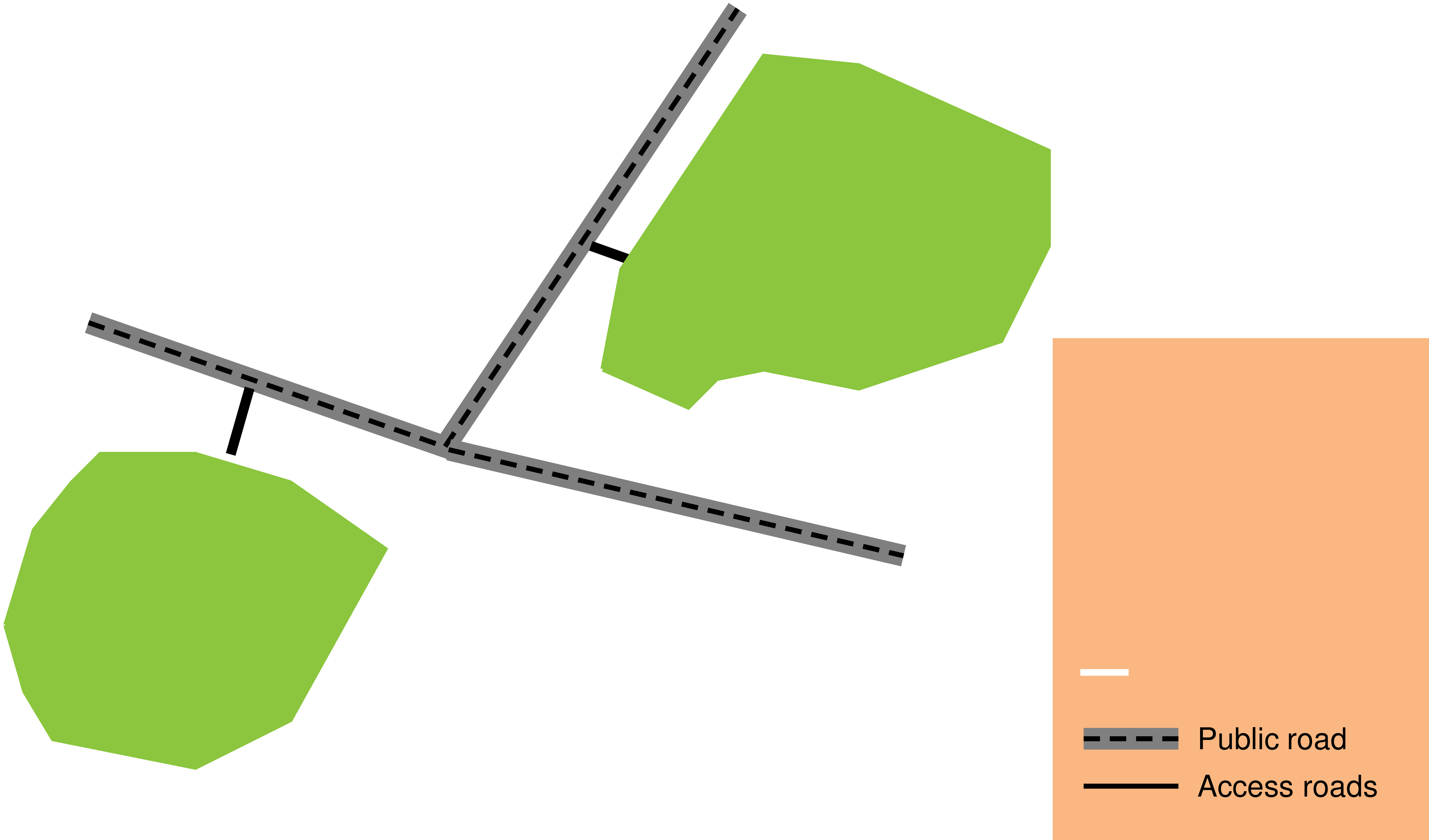
Contents

Introduction

- The forest harvest and road construction planning problem consists fundamentally of managing land designated for timber production.
- We tackle a tactical, medium range planning problem:
  - define specific cutting units to harvest in each period and,
  - the road network to access these units.
- At this level, road building leads to about 40 % of operating costs.
- We wish to consider two levels of decisions and uncertainty:
  - Strategic: uncertainty in timber production. Decisions: logistic network design.
  - Tactical: uncertainty in prices and demand. Decisions: timber harvesting and transport.

Forestry problem

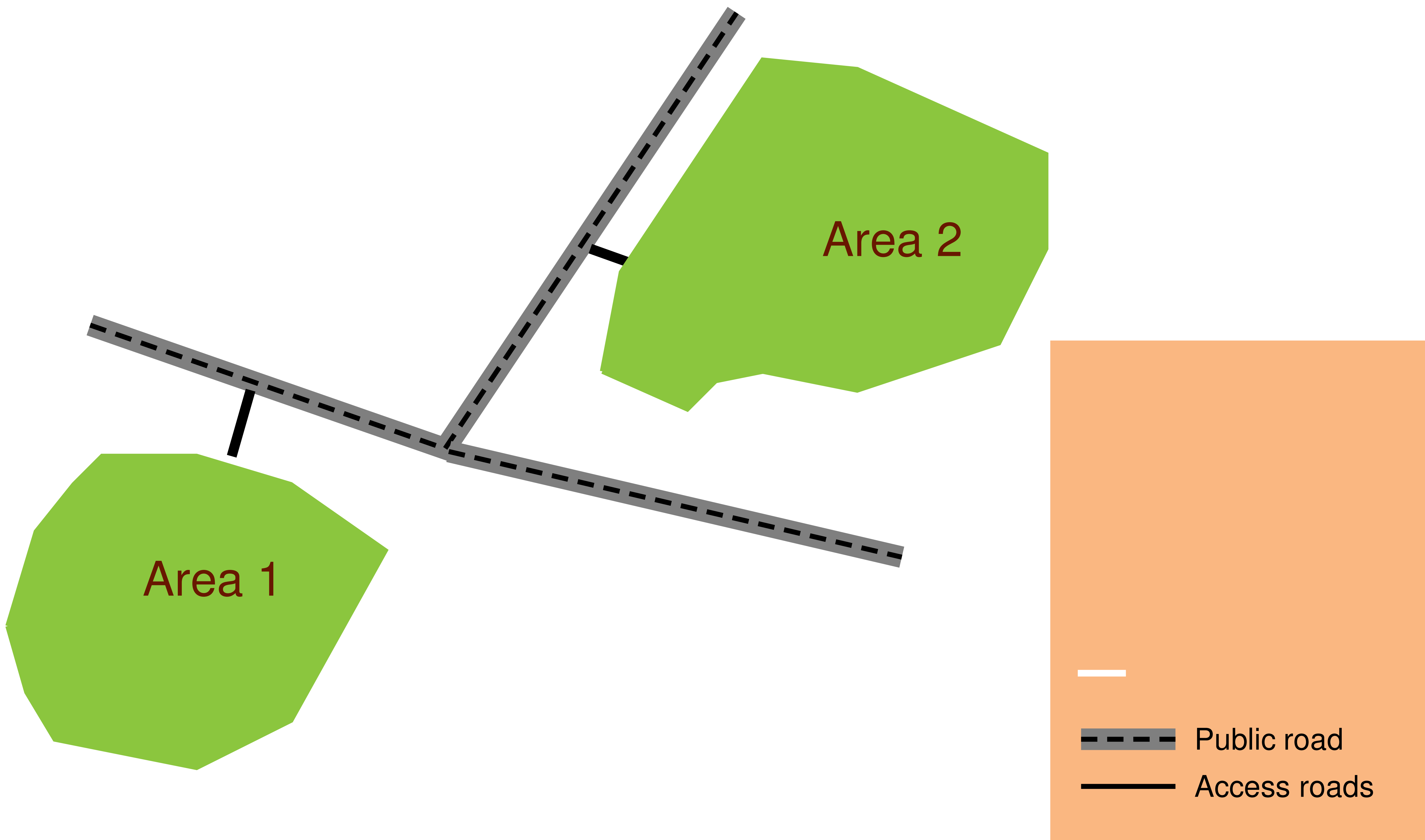
Planning horizon: 2-5 years



Supply side

Planning horizon: 2-5 years

The company owns plantation lands divided into areas



Supply side

Planning horizon: 2-5 years

Each area is divided into different homogeneous stands
(age of trees, soil quality of land and volume available per hectare).

All areas are planted with pine trees (22 to 28 years)

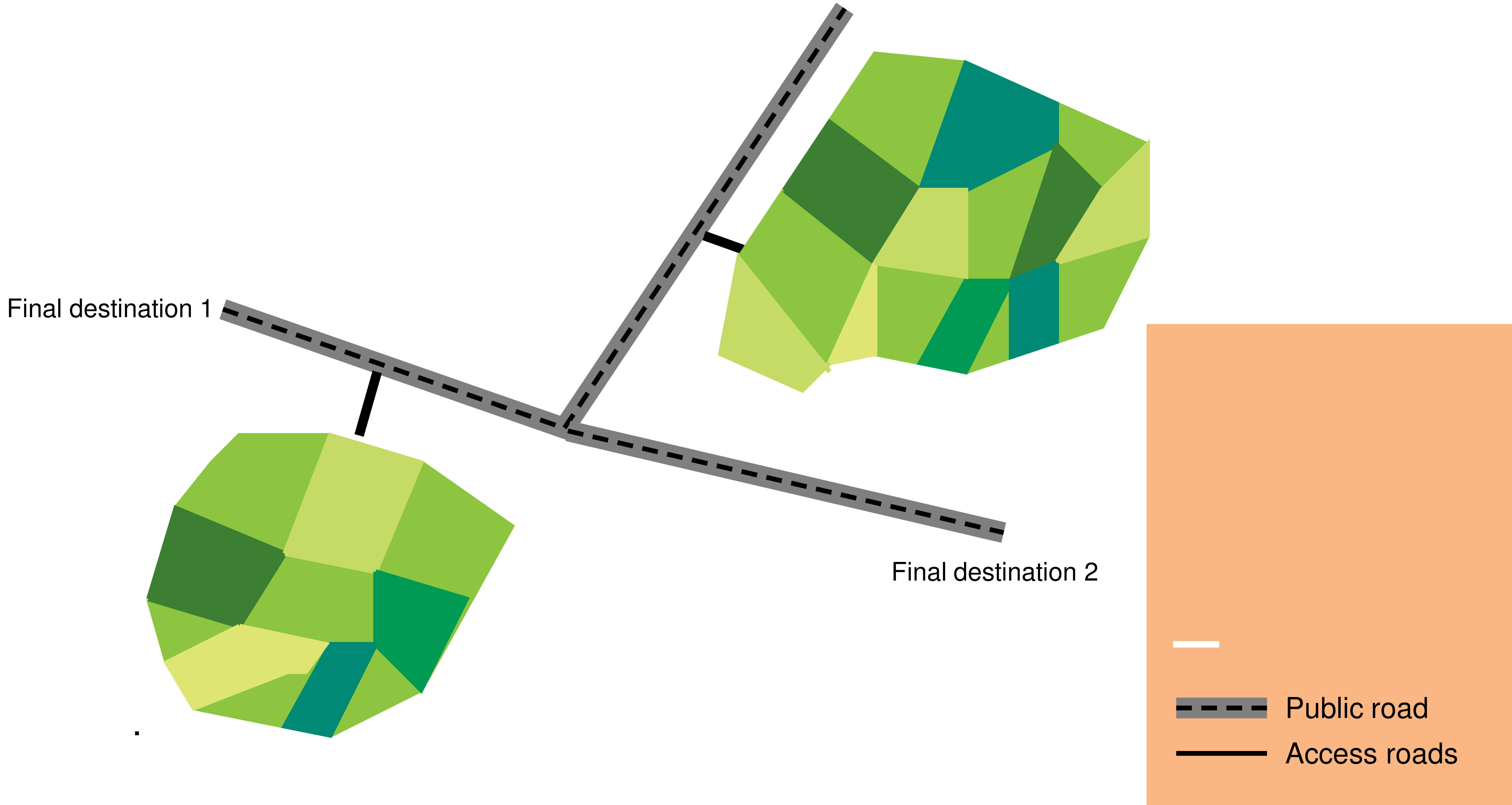
Demand side: Markets

Planning horizon: 2-5 years

Three basic aggregate products/qualities: export, sawmill, and pulp.

Higher-level quality can be used for lower-level purposes, at a loss in sale price.

For example, the pulp mill takes any type of timber, while only export quality can be exported

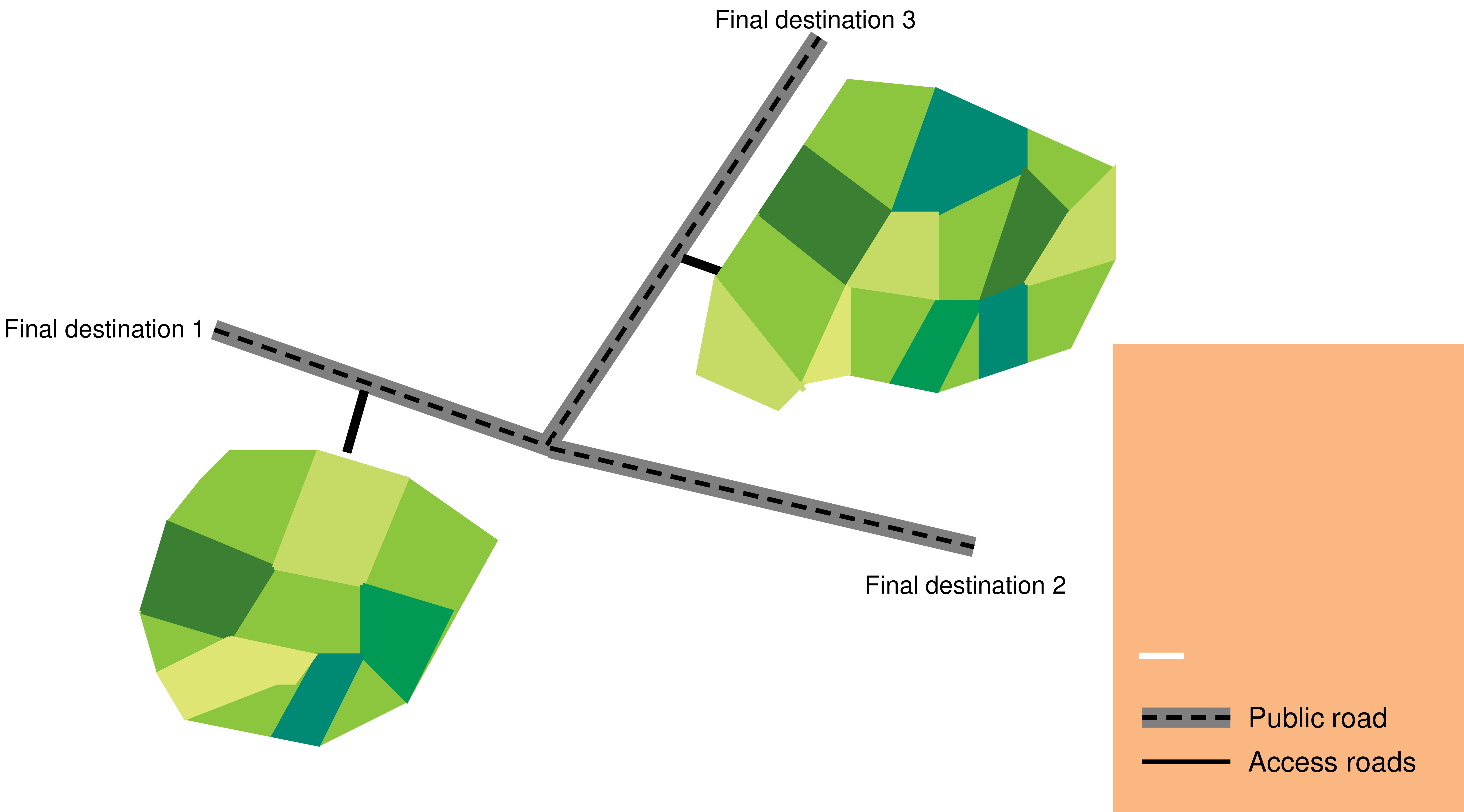


Demand side: Markets

Planning horizon: 2-5 years

Goal: to match the supply of standing timber with demands

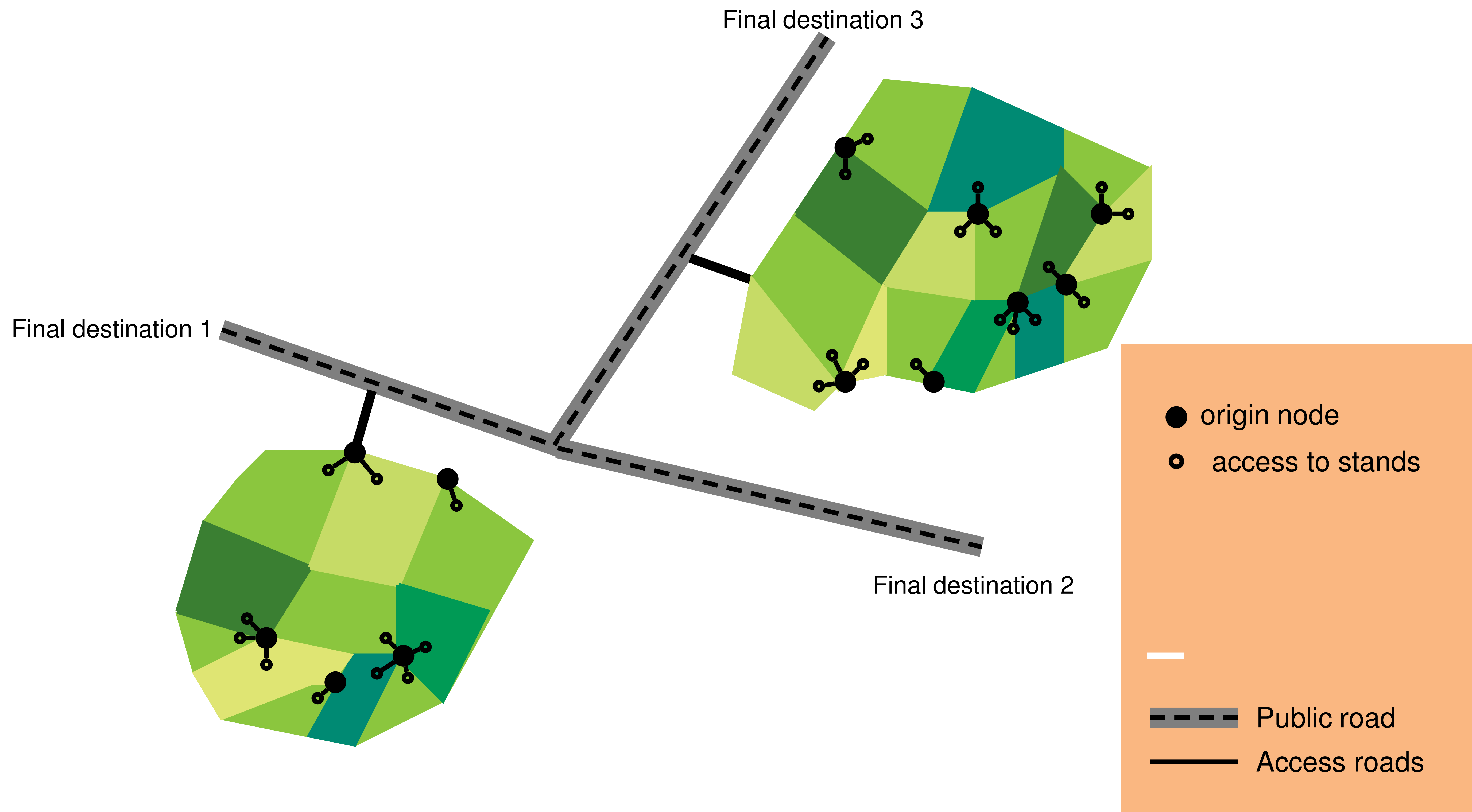
(reducing losses in revenues due to down-grading and nonprofitable additional cutting)



Logistic

Planning horizon: 2-5 years

Each stand is accessible through an origin

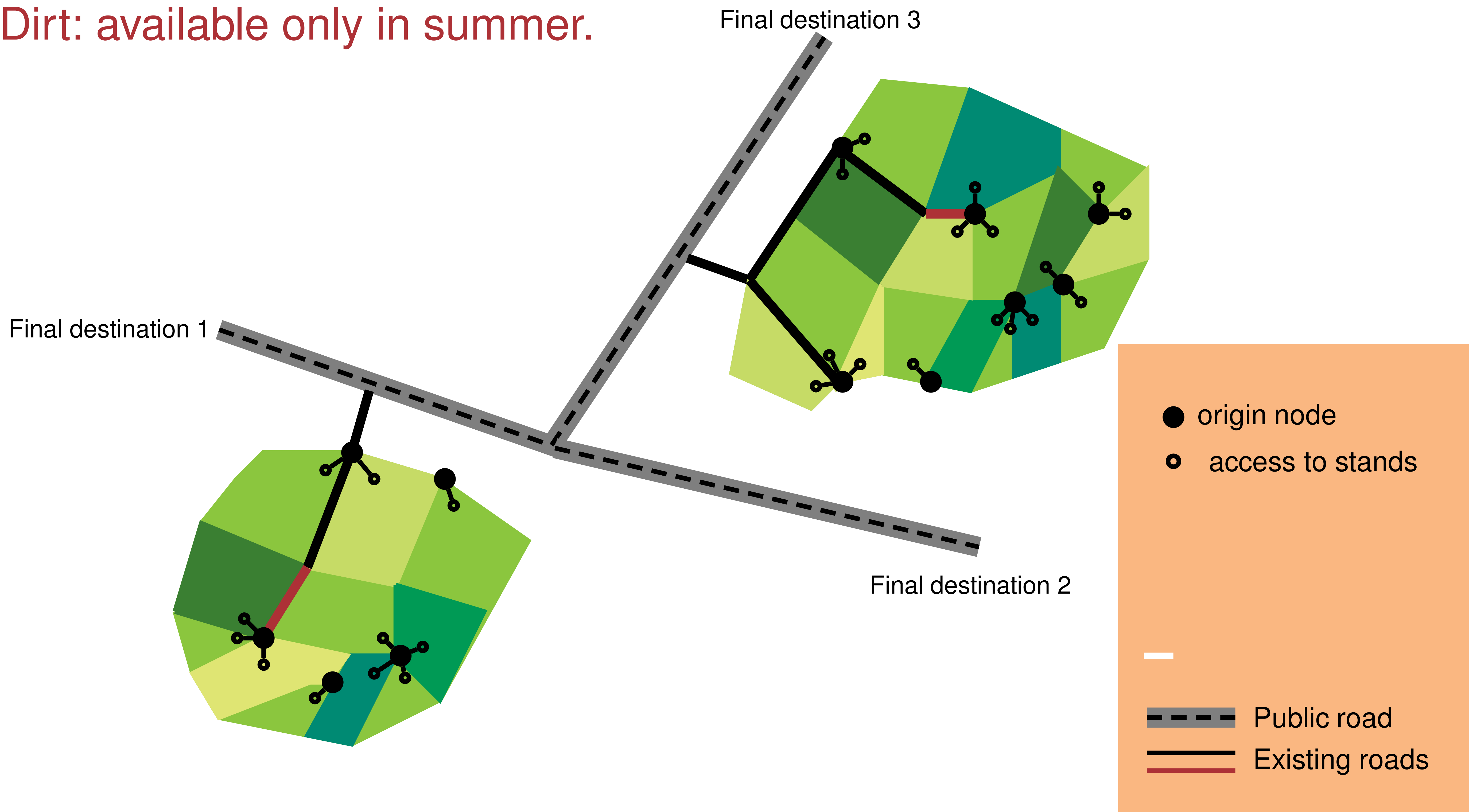


Logistic

Planning horizon: 2-5 years

There are **existing** roads inside the areas:

- Gravel: available in summer and winter.
- Dirt: available only in summer.



Logistic

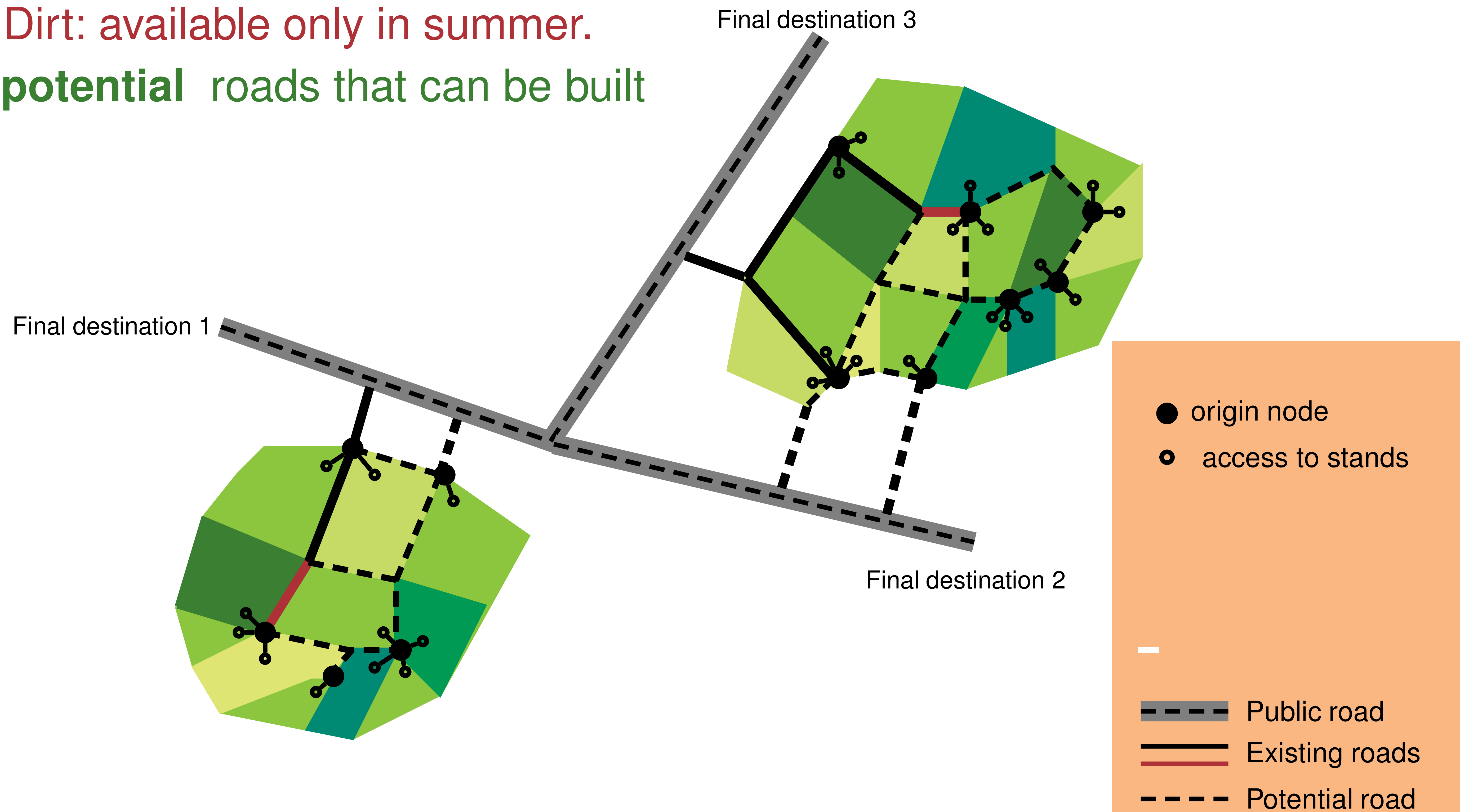
Planning horizon: 2-5 years

There are **existing** roads inside the areas:

Gravel: available in summer and winter.

Dirt: available only in summer.

and **potential** roads that can be built



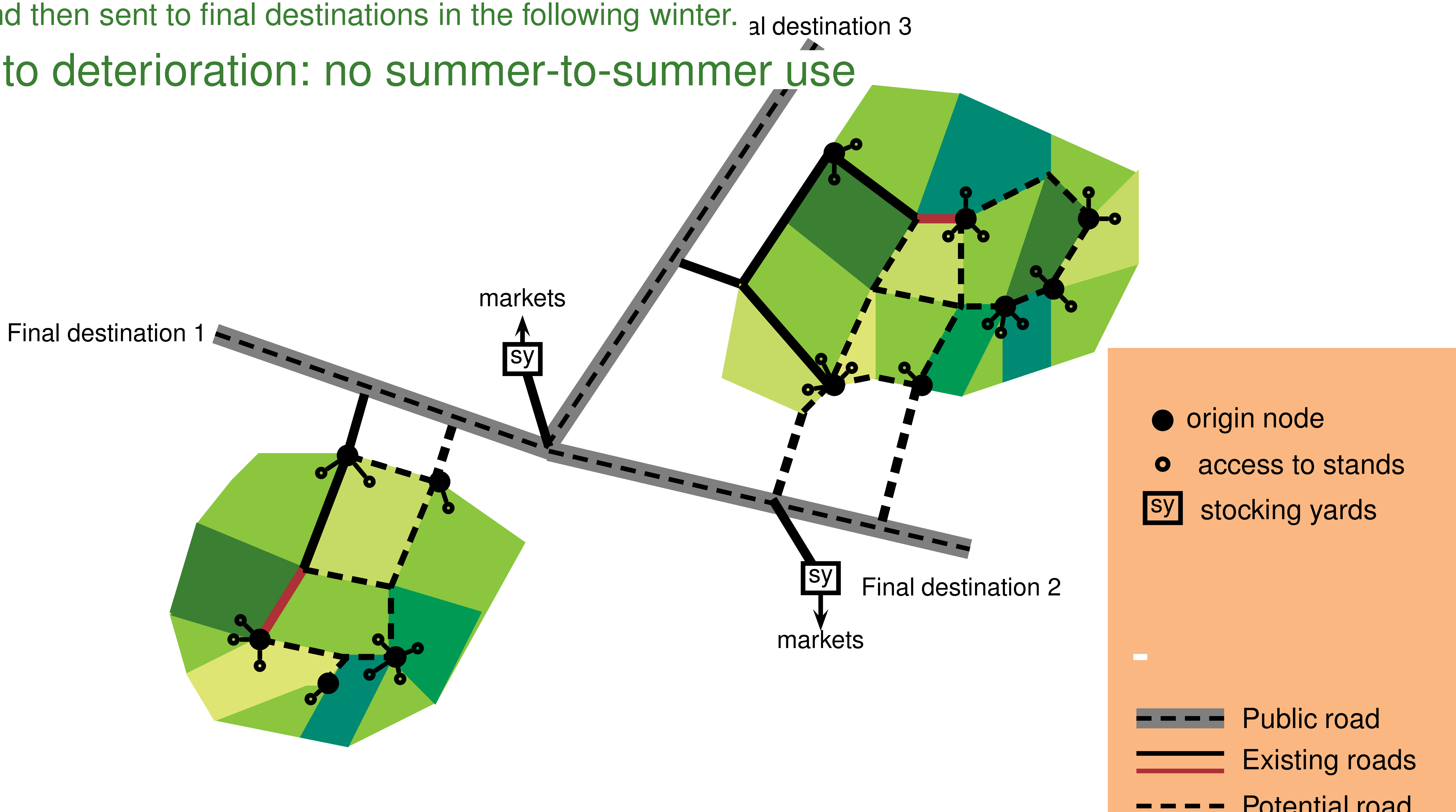
Logistic

Planning horizon: 2-5 years

There are Stocking Yards:

Timber harvested in summer can be hauled to these stocking yards and then sent to final destinations in the following winter.

Due to deterioration: no summer-to-summer use

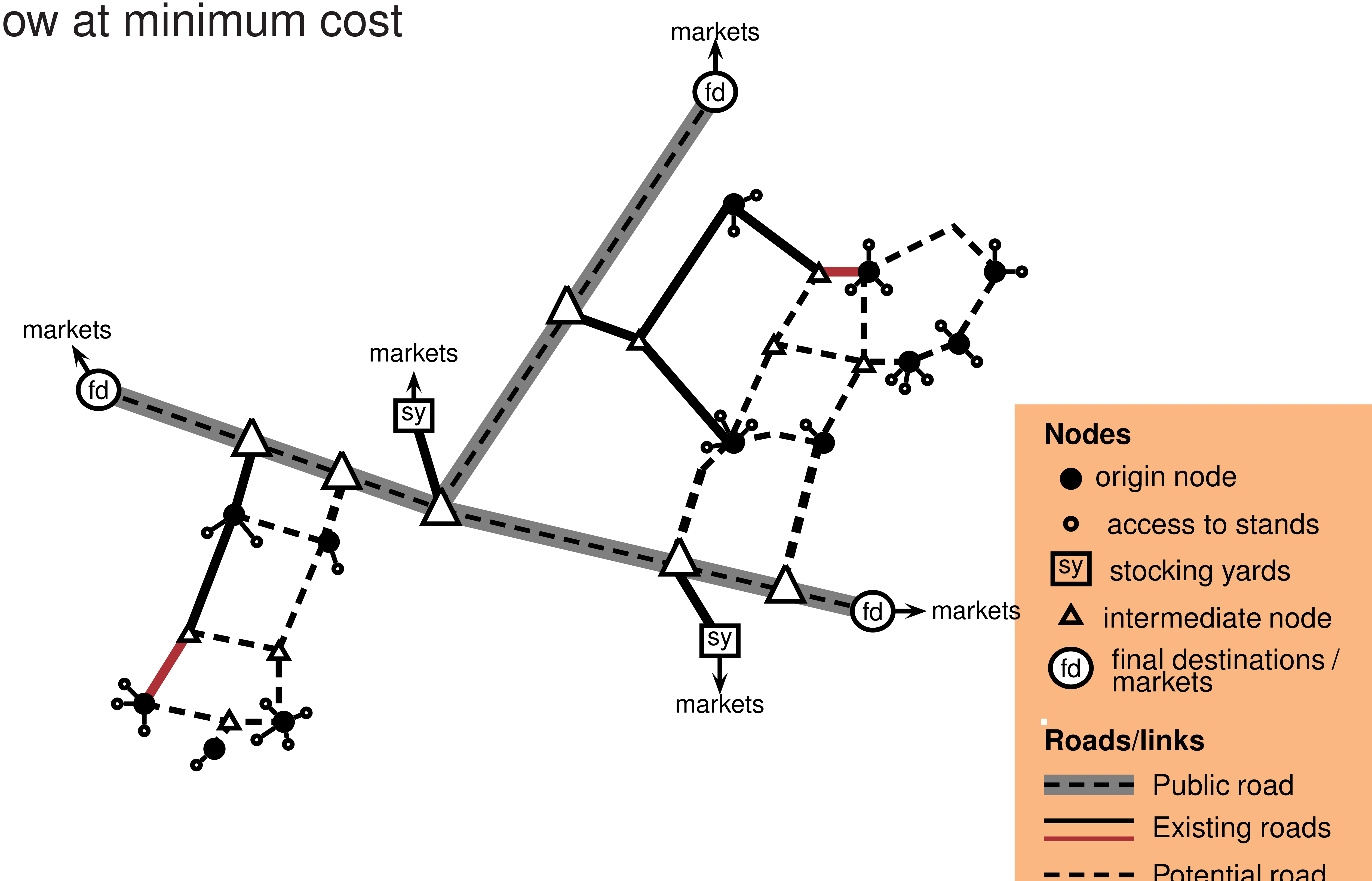


Mathematical Problem

Planning horizon: 2-5 years

Network design (dynamic and cumulative)

Network Flow at minimum cost



Deterministic approach

Objective: Maximize the profit

Income (sales of timber) **minus cost** (network building, transportation, stocking, etc.)

Decisions

- Strategic: Network design (binary):
  - Upgrade from dirt to gravel an existing road,
  - build in dirt or gravel a potential road,
  - upgrade from dirt to gravel a potential road previously built in dirt.
- Tactical: Network flow / production:
  - Sources: Stands to be harvested in each period (binary).
  - Incoming flow: Area to be harvested in each stand and period (semi-continuous).
  - Outgoing flow: Demand served at each market (continuous).
  - Path flow: Flow through the network to serve the demand of each market (continuous).

Constraints

- Strategic: Network design:
  - Decisions about new links or upgrades to gravel can be taken only in summer (it is the only period for work-load)
  - Roads in dirt are available only in summer.
  - A road is available from the period after it is built.
  - A road cannot be upgraded the same year it is built.
- Tactical: Network flow / production:
  - Bounds in the harvested area per stand and time period.
  - Flow constraints at nodes (Flow Conservation Law)
  - capacity constraints: depending of the type of road and time period
  - Stocking yards: Arrivals in summer must be equal to dispatches in the following winter.
  - Lower and upper limits in the demand per period.

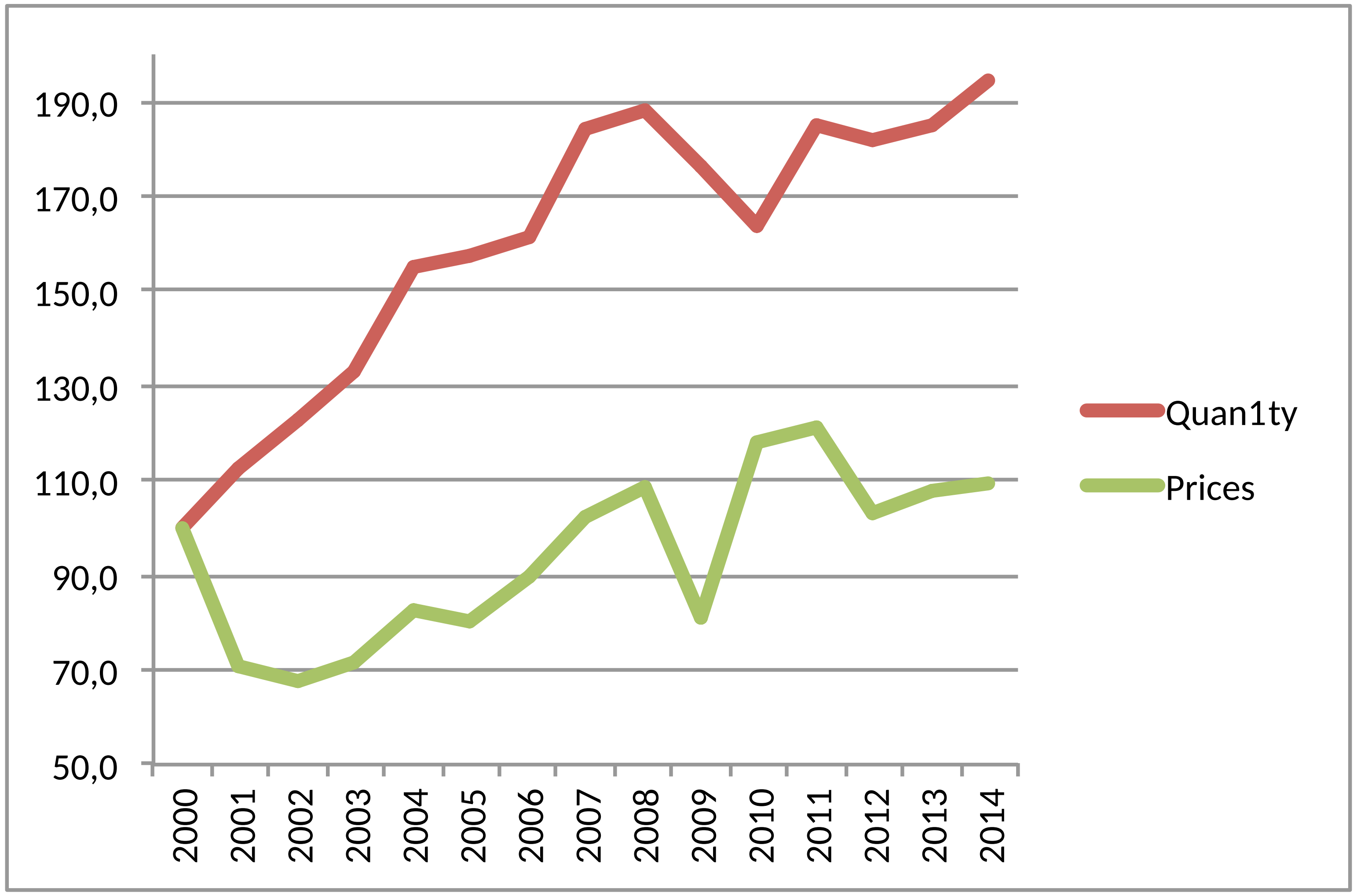
Contents

Uncertainty in (future) prices and demand

The deterministic model assumes that prices are known in advance of the planning decision.

However, wood prices and demand can vary along the planning horizon.

Year	Exports	Prices
2000	100.0	100.0
2001	112.4	70.9
2002	122.6	67.3
2003	132.8	71.9
2004	155.4	82.2
2005	157.2	80.2
2006	161.3	89.5
2007	184.4	102.4
2008	188.0	108.9
2009	176.3	81.2
2010	163.9	118.1
2011	185.0	121.3
2012	181.7	102.9
2013	185.2	107.6
2014	194.6	109.4

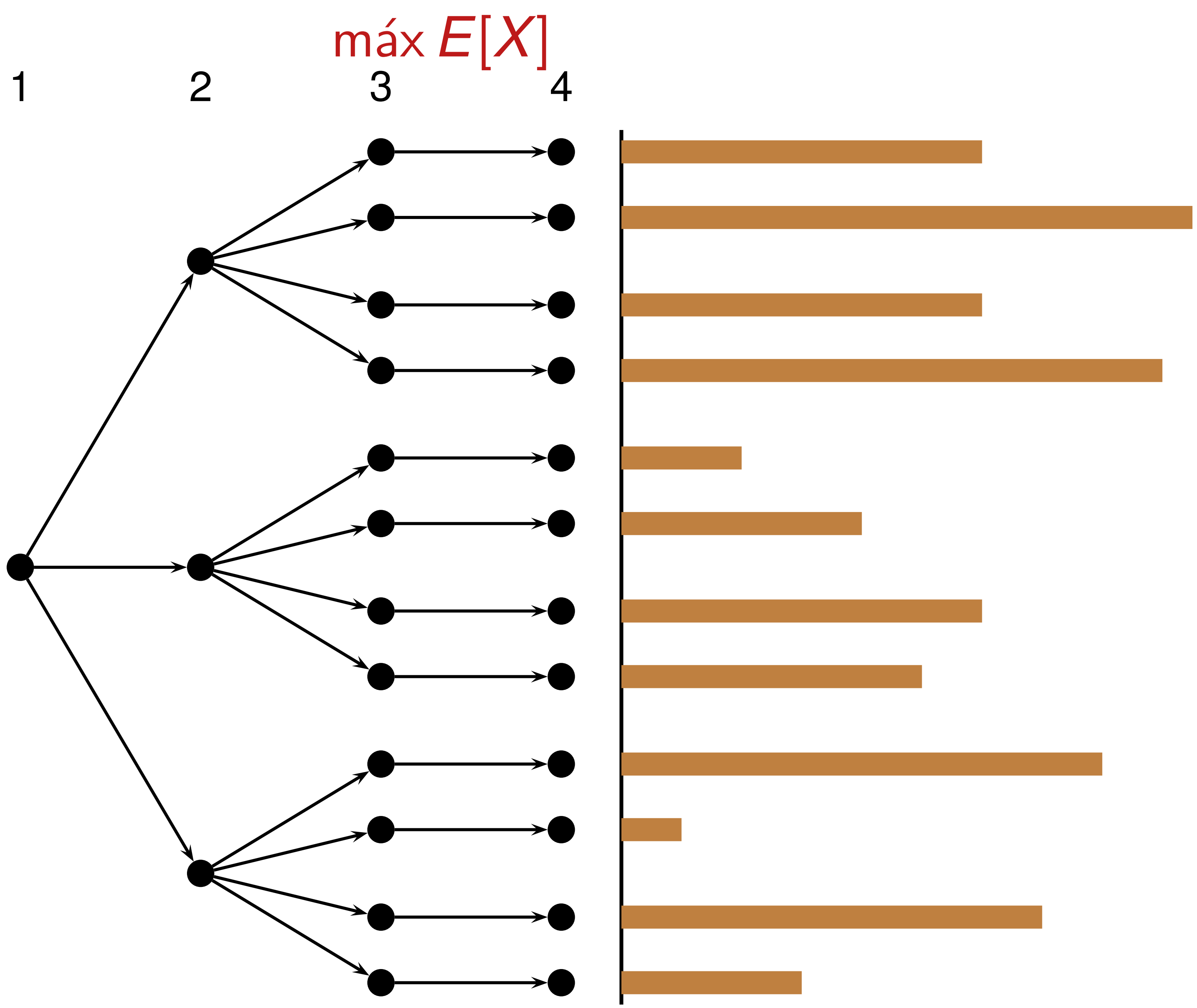


Chilean forest exports index of wood quantity and price. (base: Avr. year 2000=100)

Notice the volatility of the uncertain parameters which are therefore very difficult to predict.

Risk management and stochastic optimization

Traditional approach: optimization of the expected profit



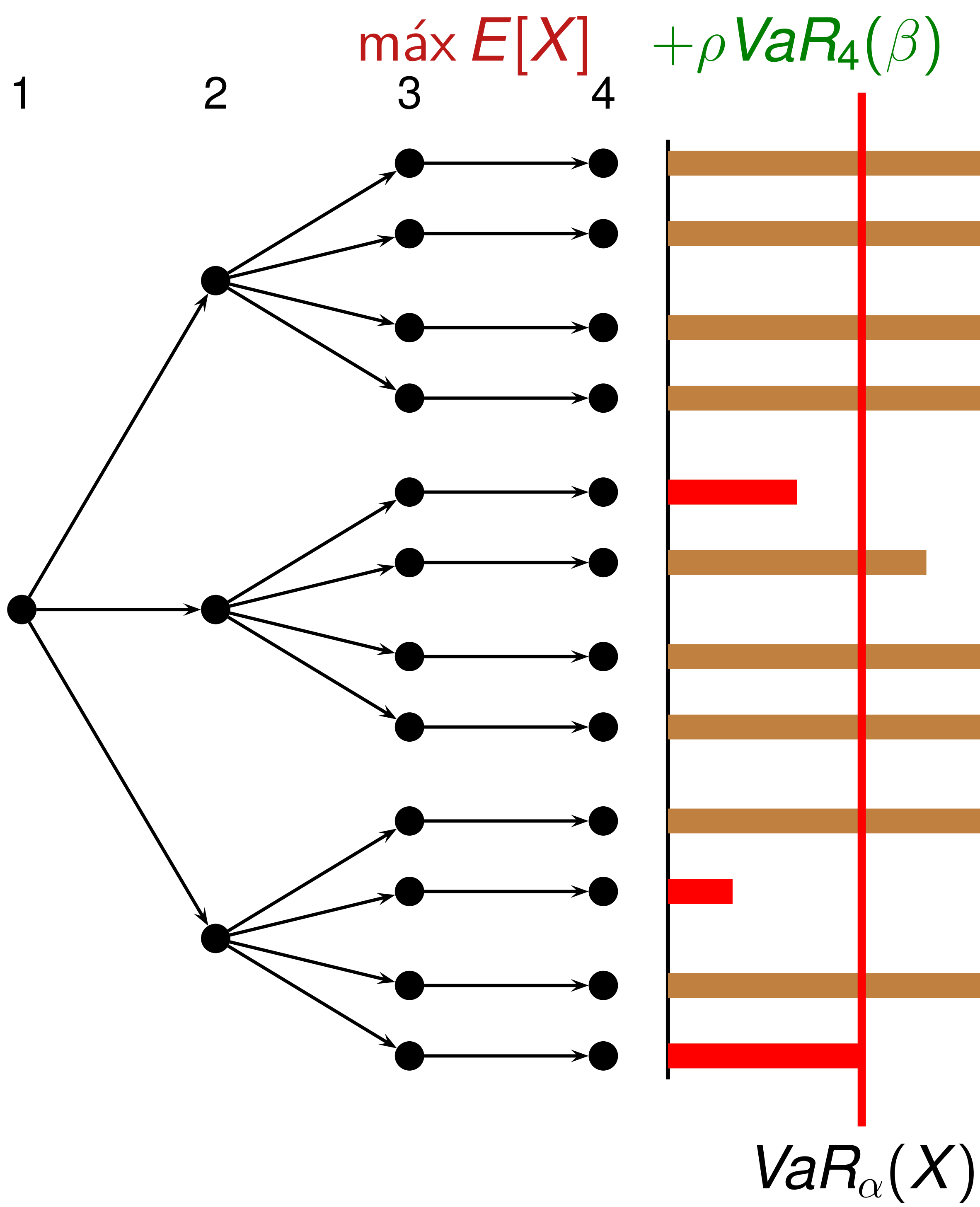
RN: Risk Neutral (Dantzig, 1955)

- Time consistent.
- No take into account the tails of the distribution.

Risk management and stochastic optimization

Traditional approach: optimization of the expected profit

and risk control in the last stage



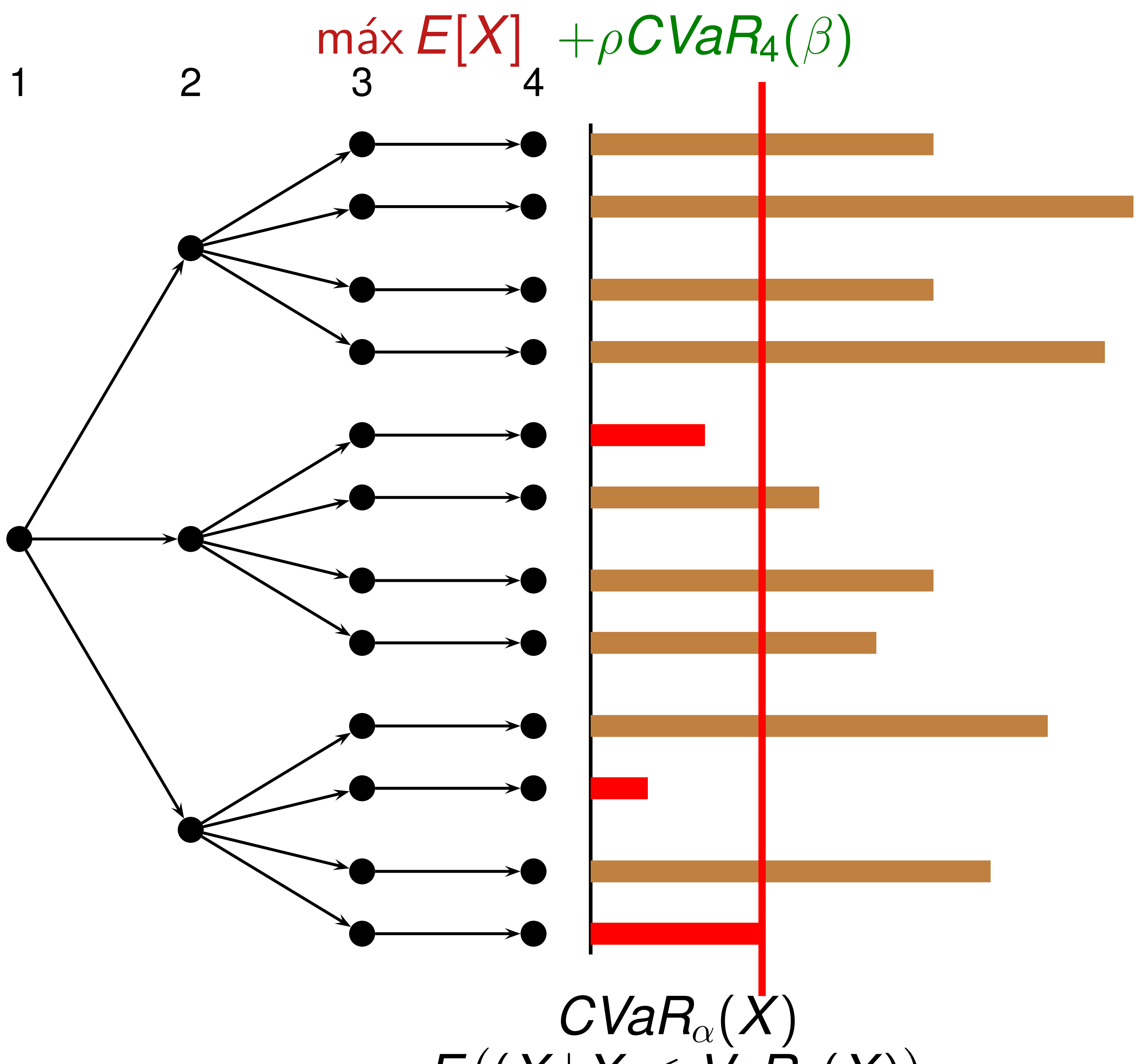
VaR: Value at Risk

- No coherent risk measure.
- It does no take into account the tails of the distribution.

Risk management and stochastic optimization

Traditional approach: optimization of the expected profit

and risk control in the last stage



CVaR:
Conditional Value at Risk

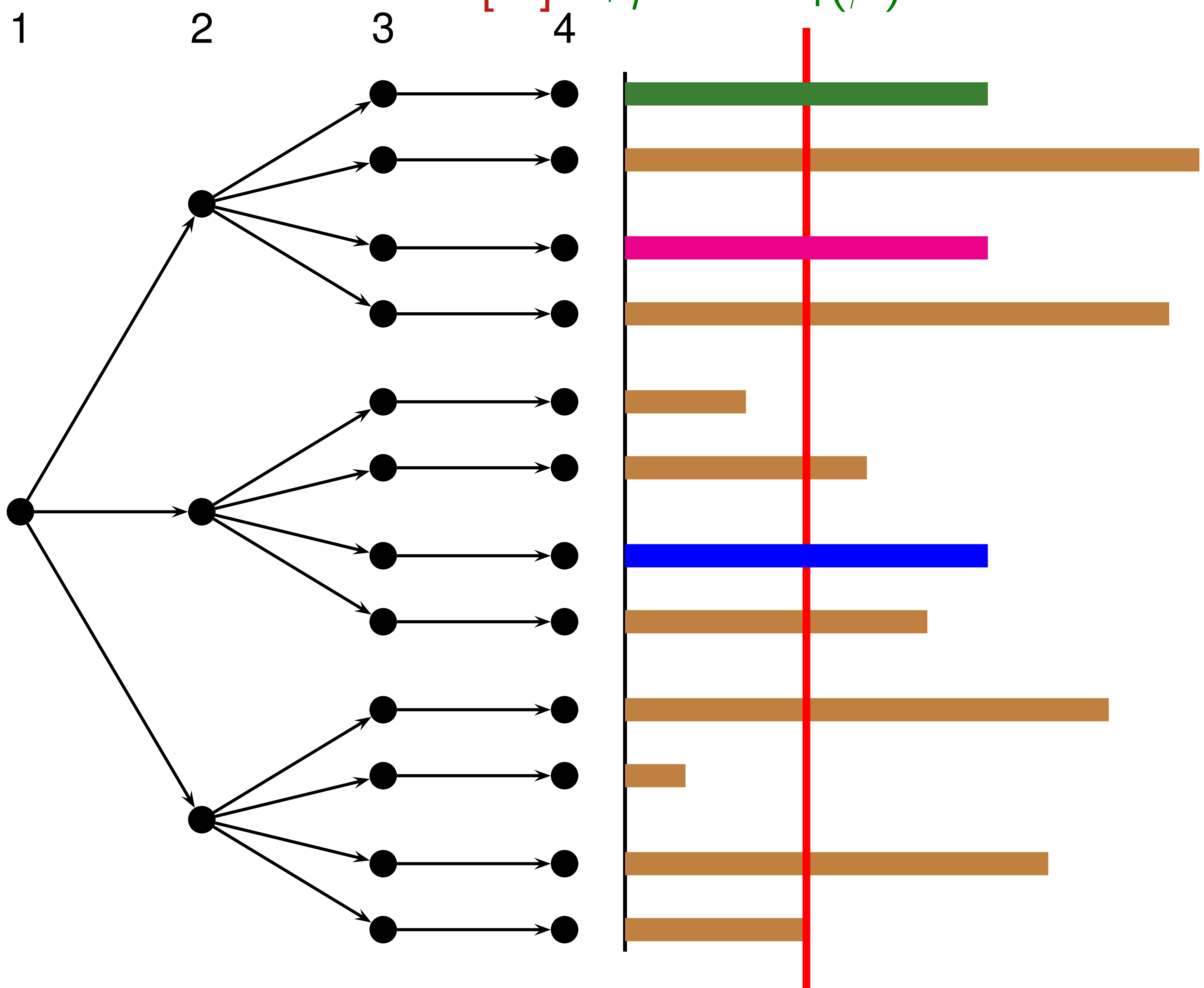
- Coherent risk measure.
- No time consistent (the solution changes if the model is reoptimiced along the time horizon).
- It does no take into account risk at intermediate periods.

Risk management and stochastic optimization

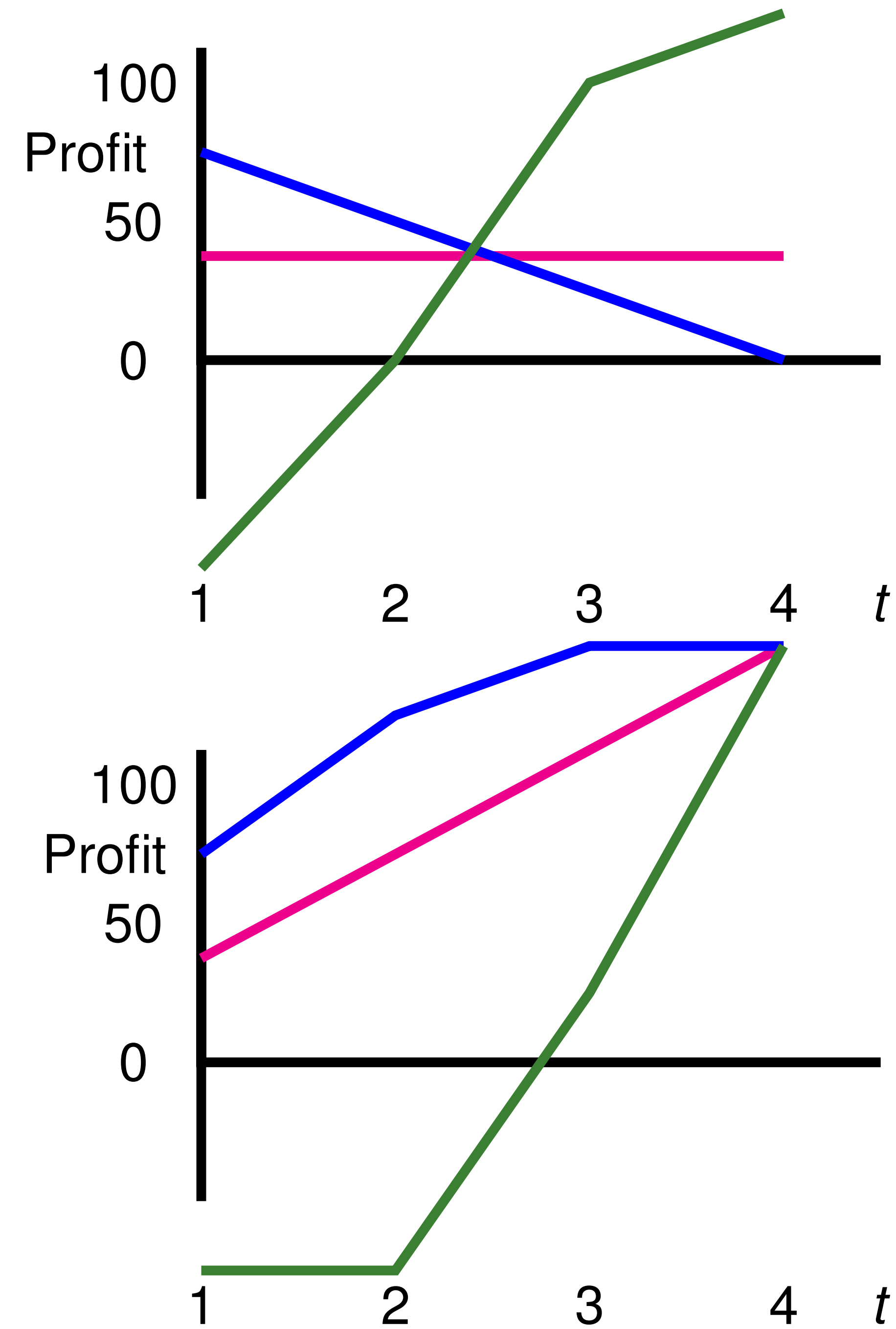
Traditional approach: optimization of the expected profit

and risk control in the last stage

$\max E[X] \quad +\rho CVaR_4(\beta)$



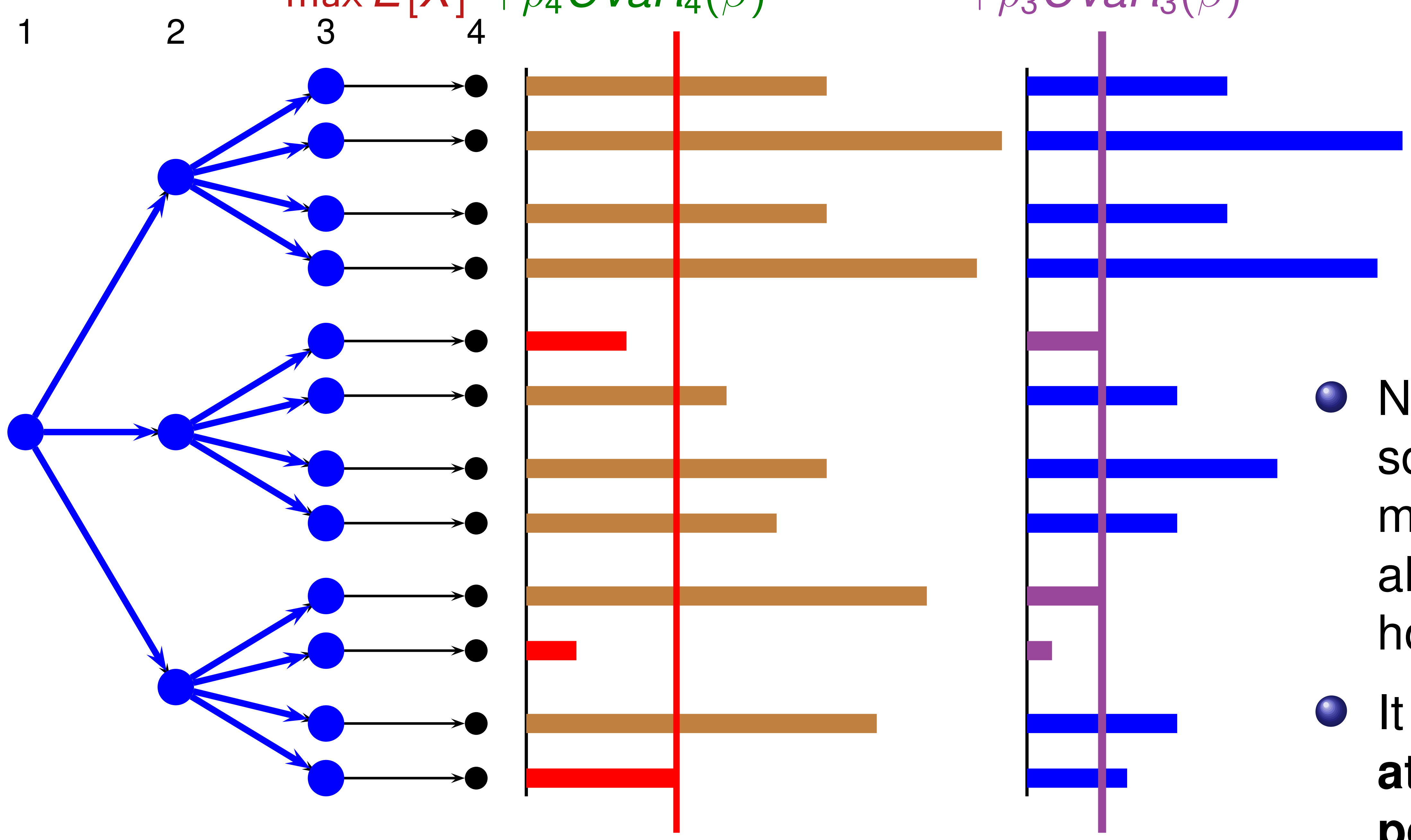
$CVaR_\alpha(X)$



Risk management and stochastic optimization

Risk control at intermediate stages (not only at the last one)

máx $E[X]$ $+\rho_4 CVaR_4(\beta)$ $+\rho_3 CVaR_3(\beta)$



TCVaR:

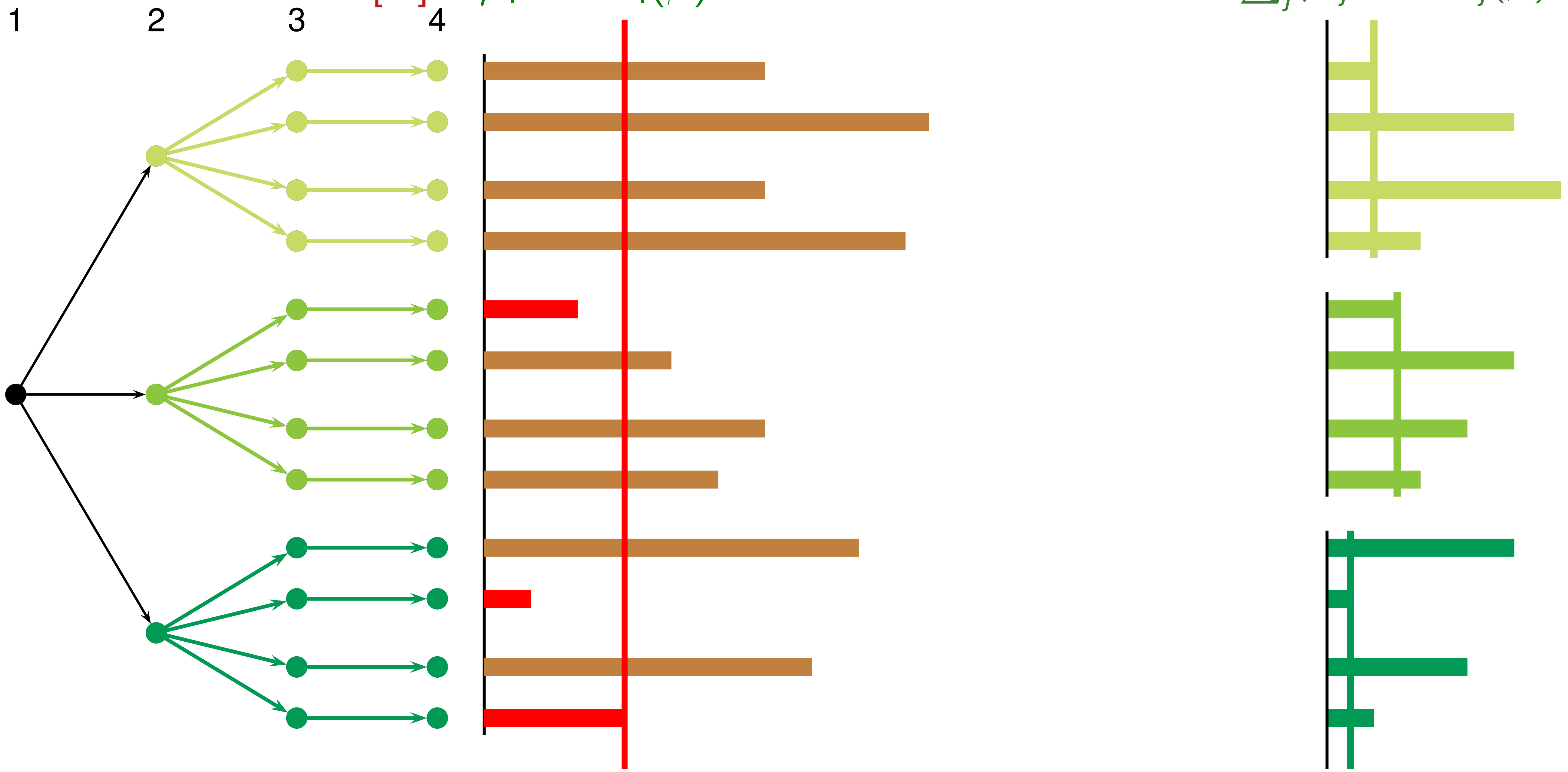
- No time consistent (the solution changes if the model is re-optimized along the time horizon).
- It take into account risk **at intermediate periods**

TCVaR

Risk management and stochastic optimization

Risk control at intermediate nodes (not only at the root)

$\text{máx } E[X] \; + \rho_4 CVaR_4(\beta)$ $\quad + \sum_j \rho_{2j} CVaR_{2j}(\beta)$



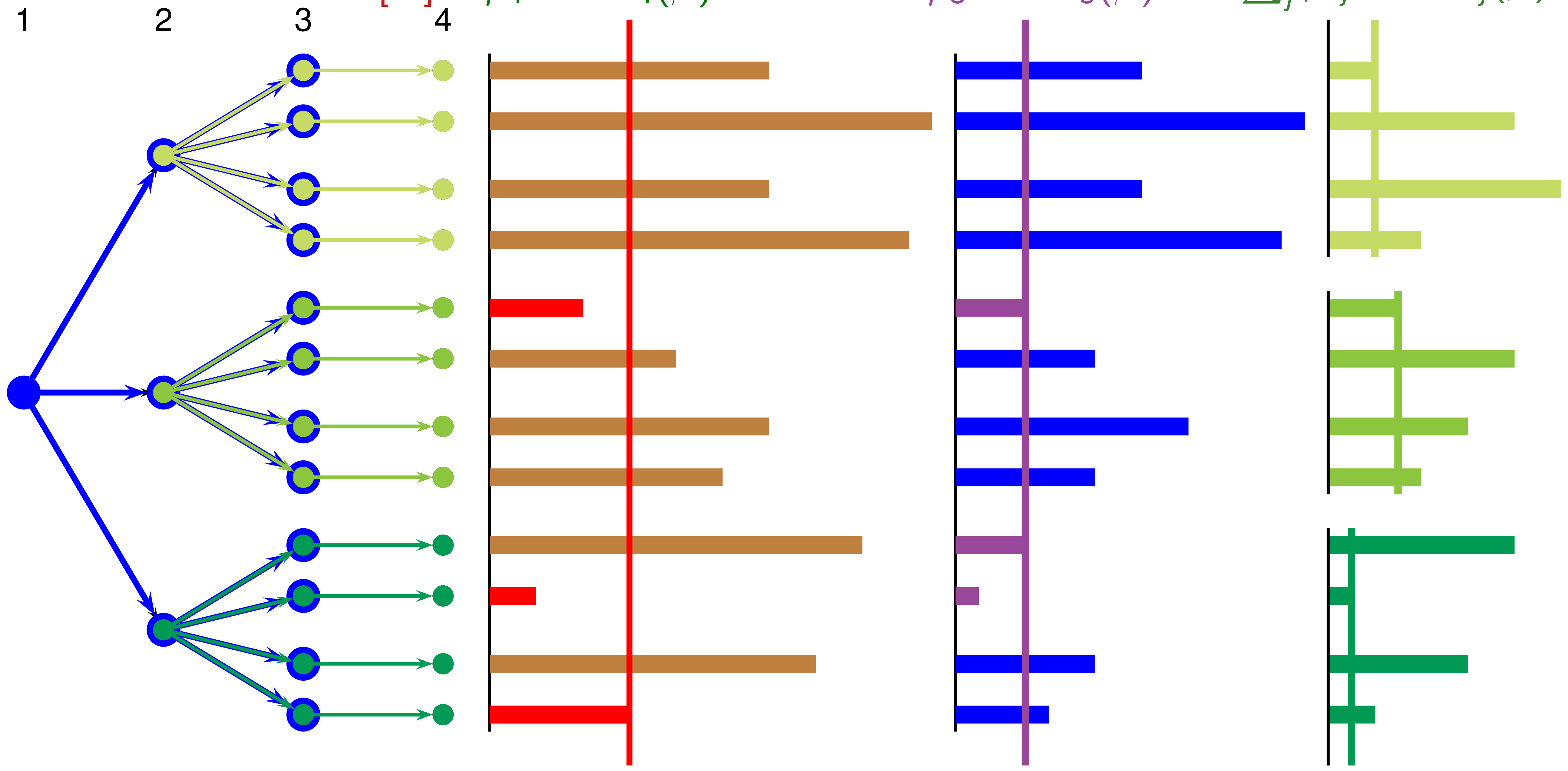
ECVaR

Risk management and stochastic optimization

Risk control at intermediate stages (not only at the last one)

and at intermediate nodes (not only at the root)

$\max E[X] + \rho_4 CVaR_4(\beta) \quad + \rho_3 CVaR_3(\beta) \quad + \sum_j \rho_{2j} CVaR_{2j}(\beta)$



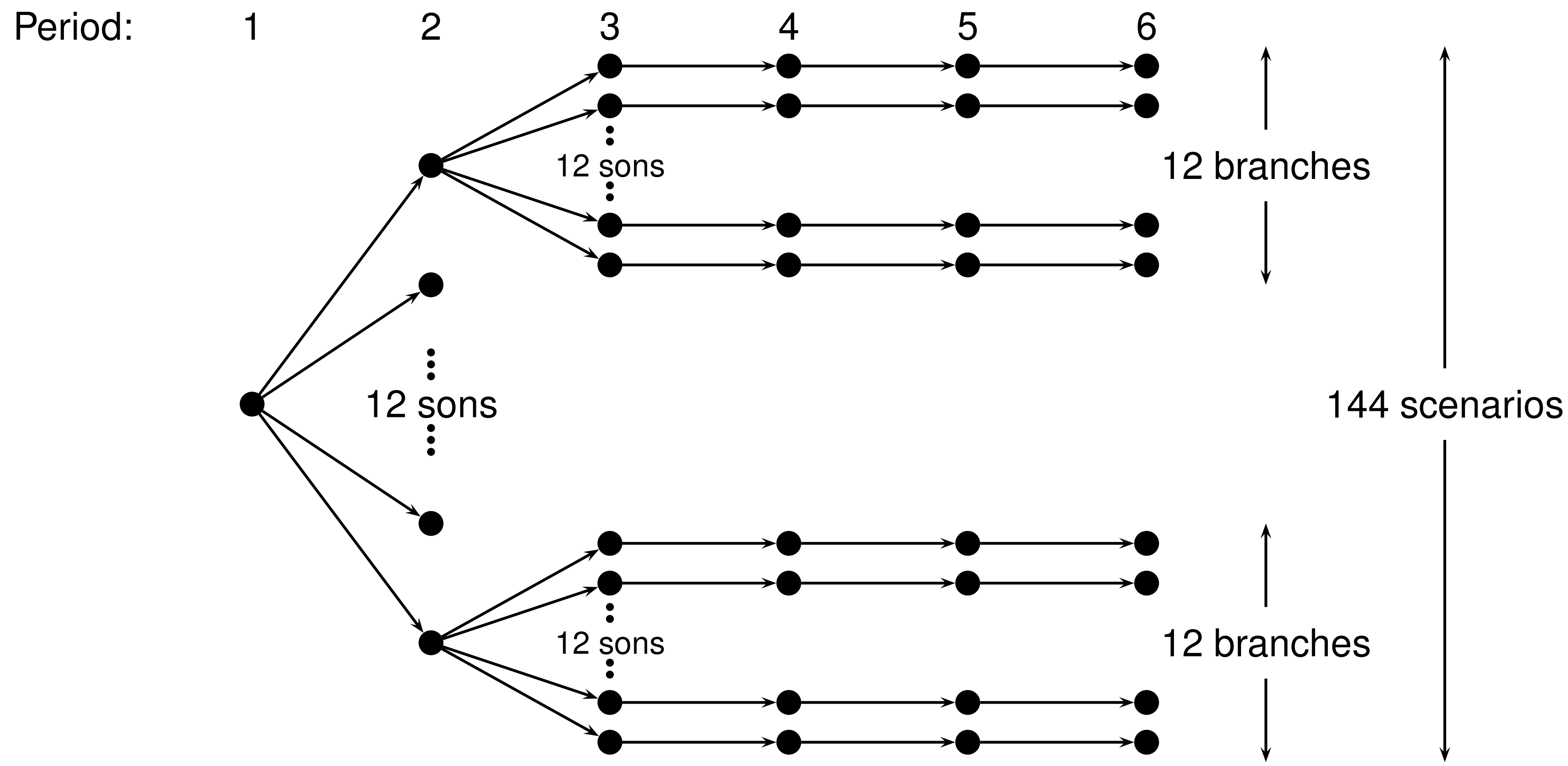
MCVaR (mixture of TCVaR and ECVaR)

Case description

- The forest company, Forestal Millalemu, owns 21 areas, geographically separated and connected to markets

Ins.	na	$\mathcal{S}$	$\mathcal{C}$	$\mathcal{I}$	$\mathcal{L}^P$	$\mathcal{L}^{Ed}$	$\mathcal{L}^{Eg}$	Ha
i1	2	11	0	15	7	7	0	627.4
i2	3	14	0	20	7	10	0	694.0
i3	2	21	0	33	8	16	9	216.1

- 3 stages: 6 periods (three years and summer/winter):



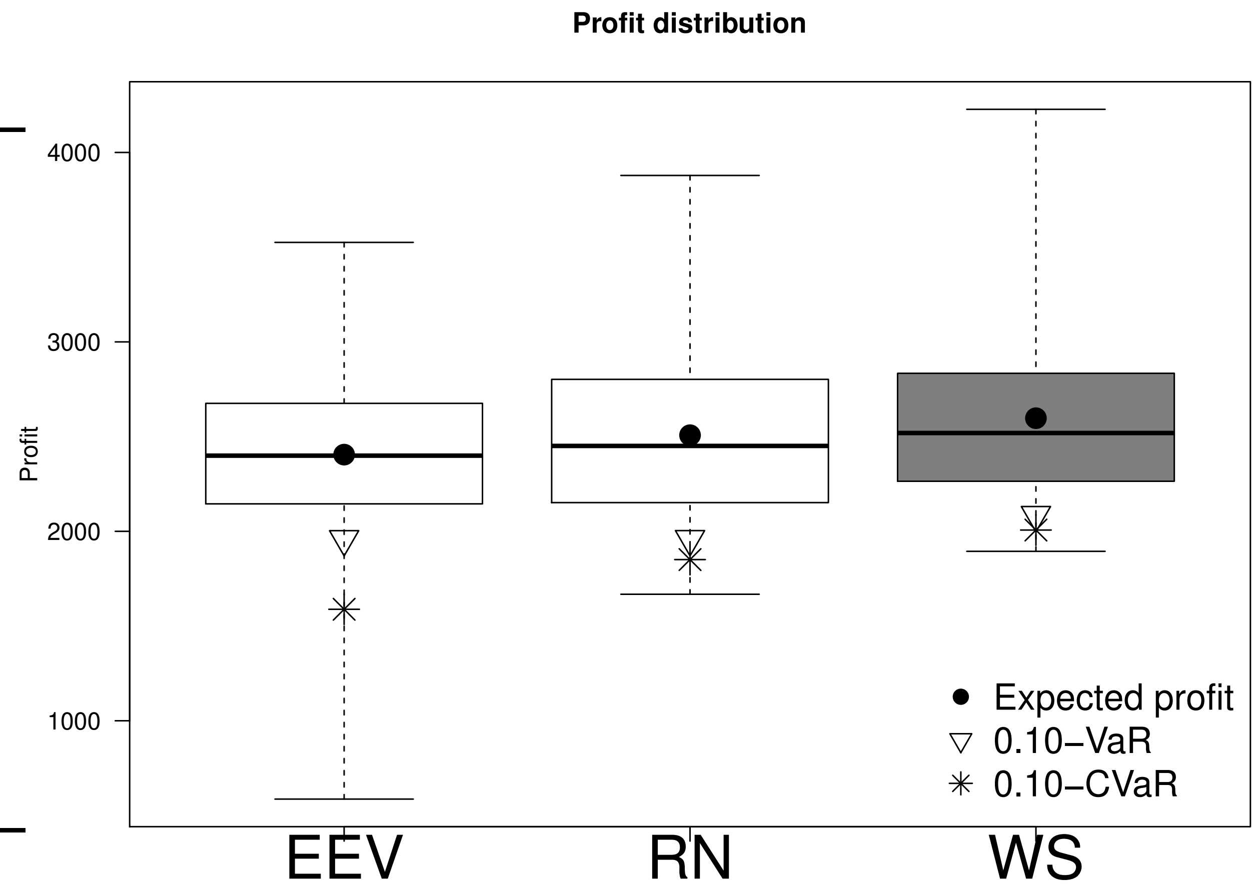
Models dimensions

	Deterministic			RN Stochastic model				
Ins.	m	nc	$n01$	m	nc	$n01$	Ω	$\mathcal{N}$
i1	1531	2399	220	158919	234070	22880	144	589
i2	1916	3311	268	198288	323598	27592	144	589
i2	2365	4063	350	245806	397136	35825	144	589

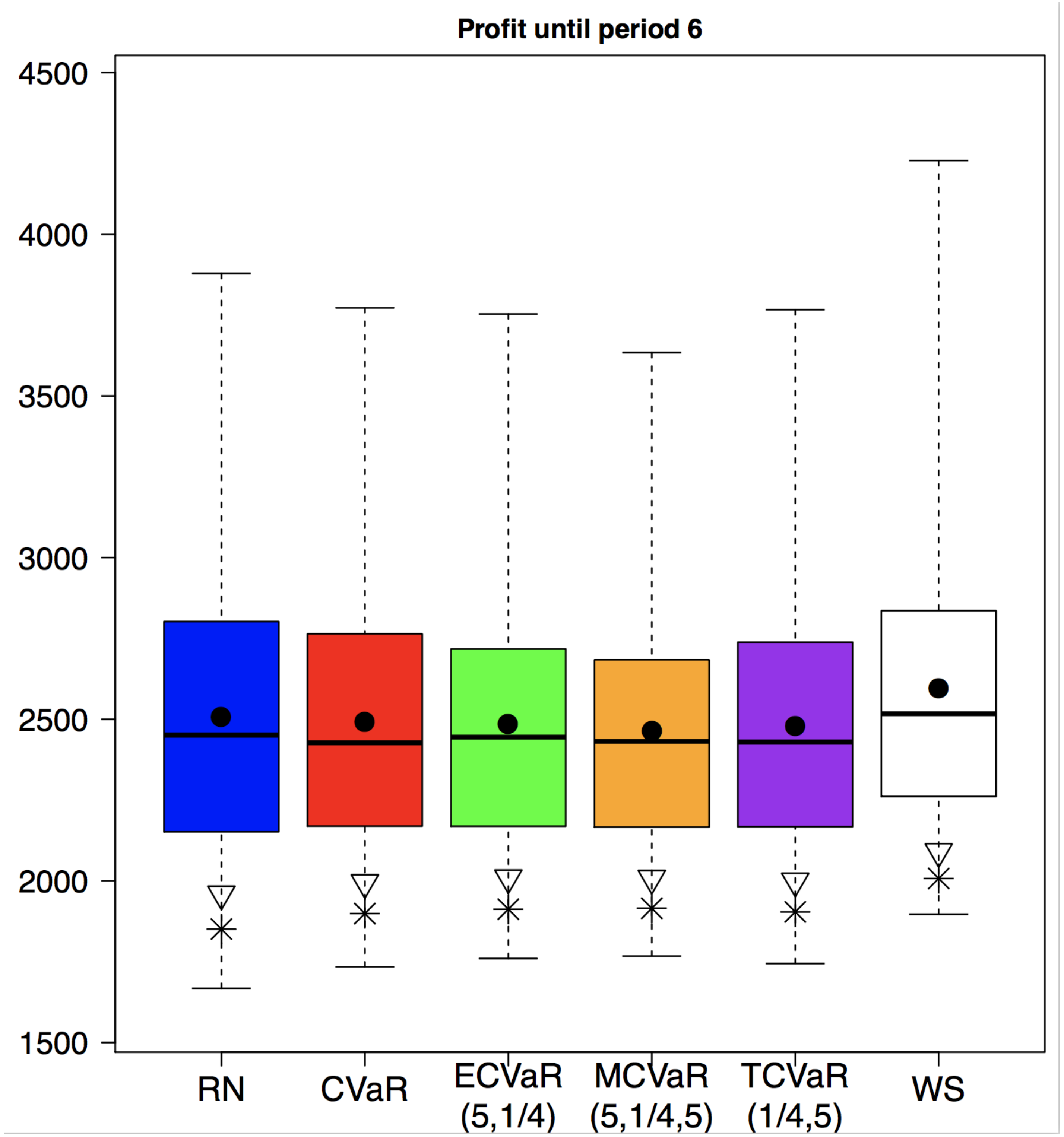
- Computer details:
  - Gams 24.2.2 and Cplex 12.6.
  - Two Intel Xeon 6 cores 2.3 Ghz, 64GRAM
- Optimality gap set to 2 %.
- Some models need up to 20 hours!!!

Results. Risk neutral versus deterministic

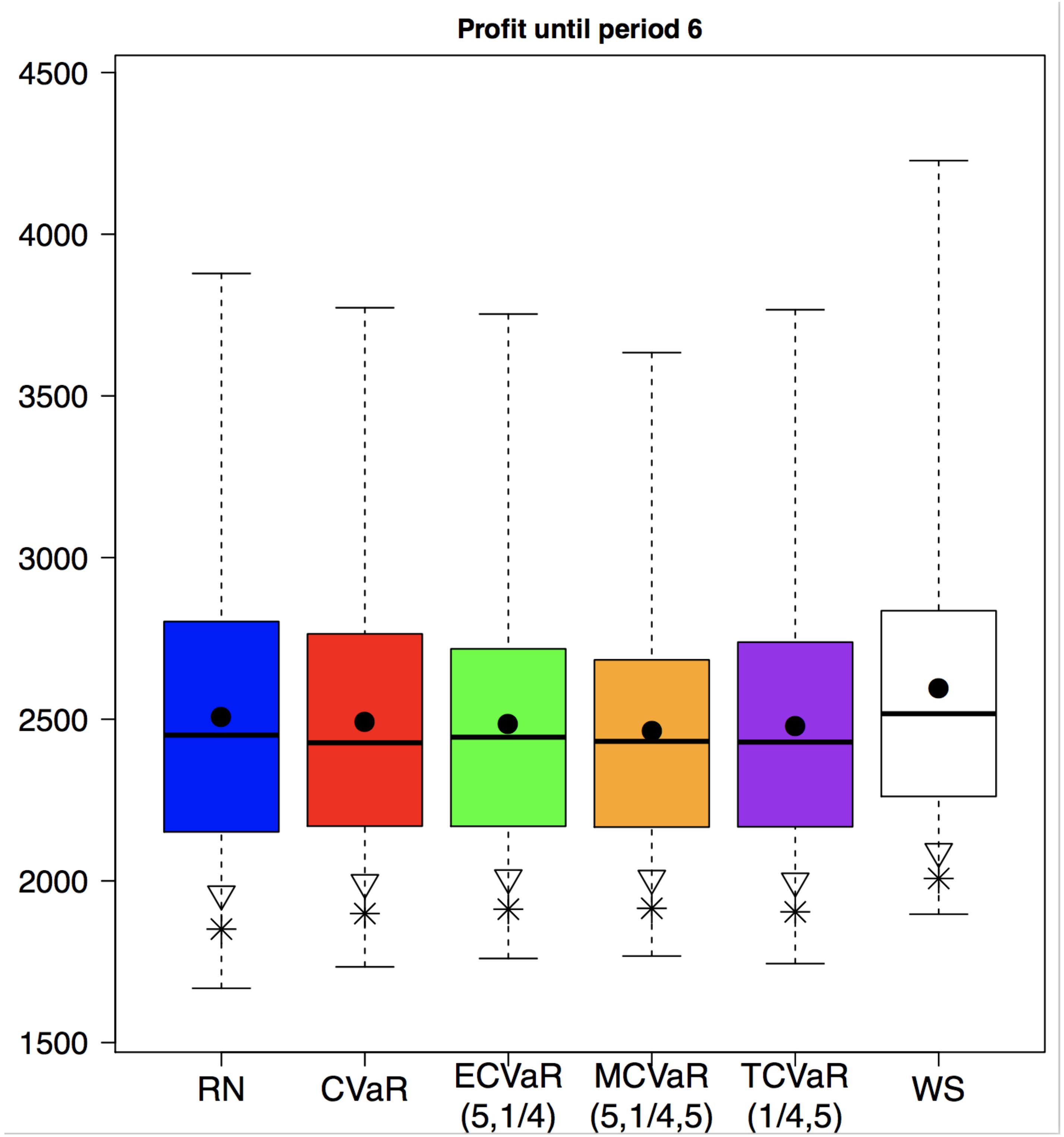
	EEV	RN	WS
Solution value	2415	2475	2553
Greatest scen.	3525	3879	4228
Median	2399	2451	2519
c-VaR	1959	1959	2087
c-CVaR	1589	1851	2006
Smallest scen.	587	1668	1894
CPU time (secs.)	11	34156	123

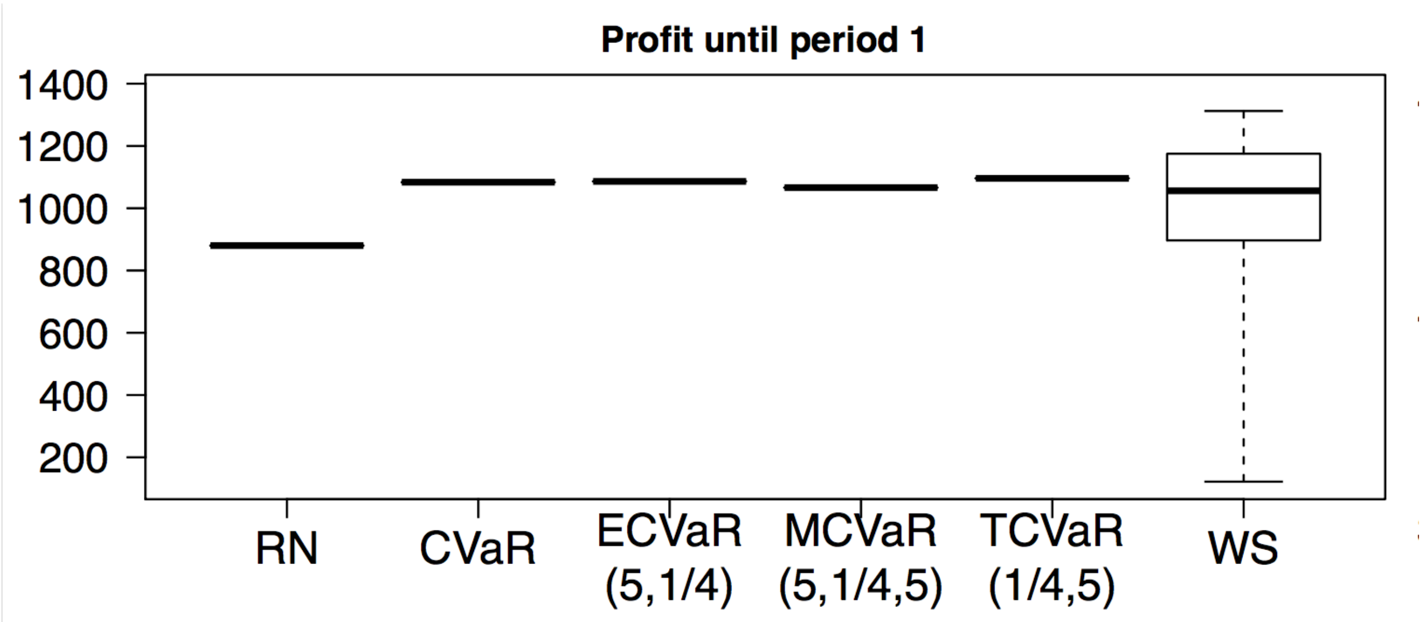


Computational results
Profit distribution

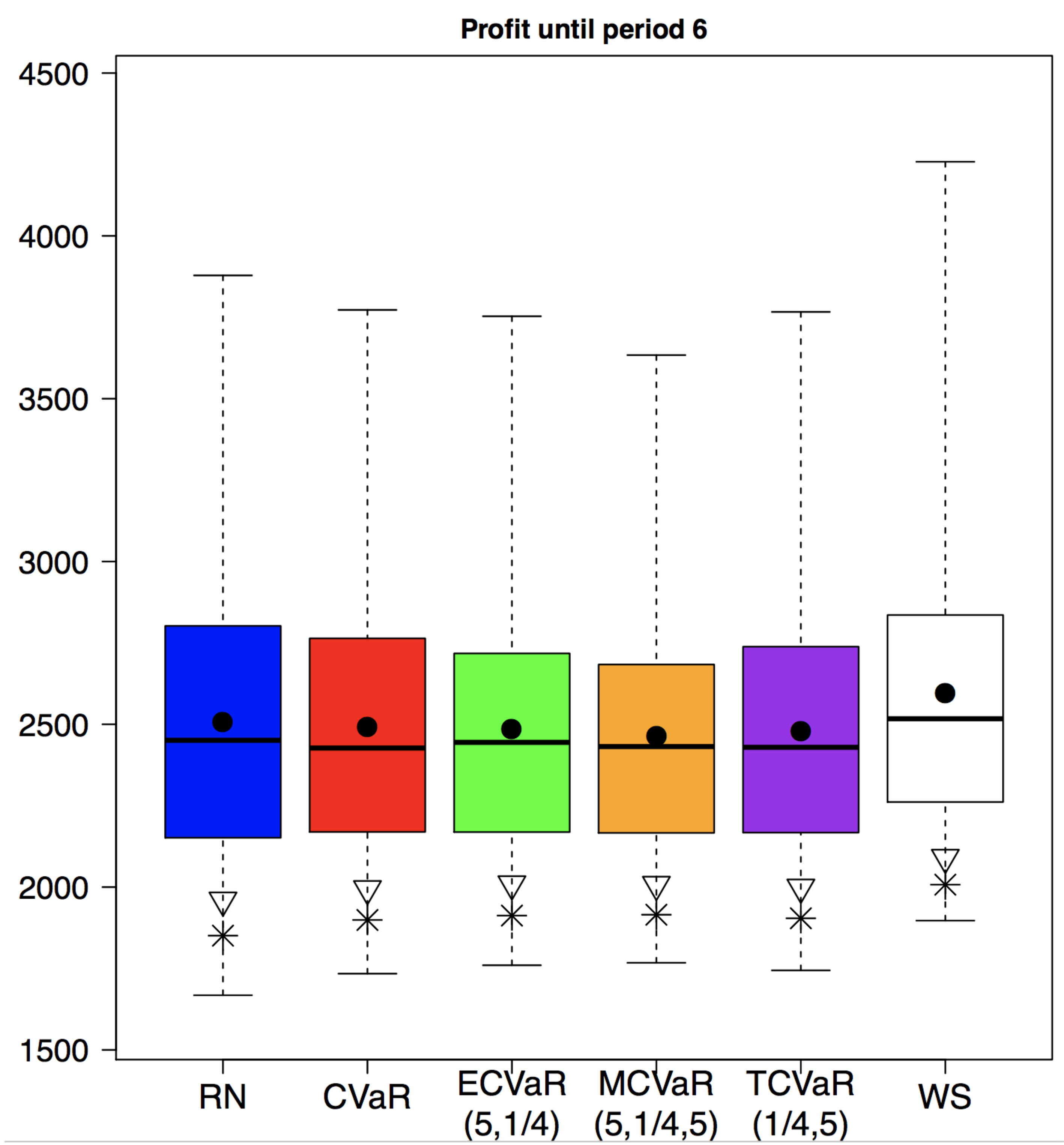


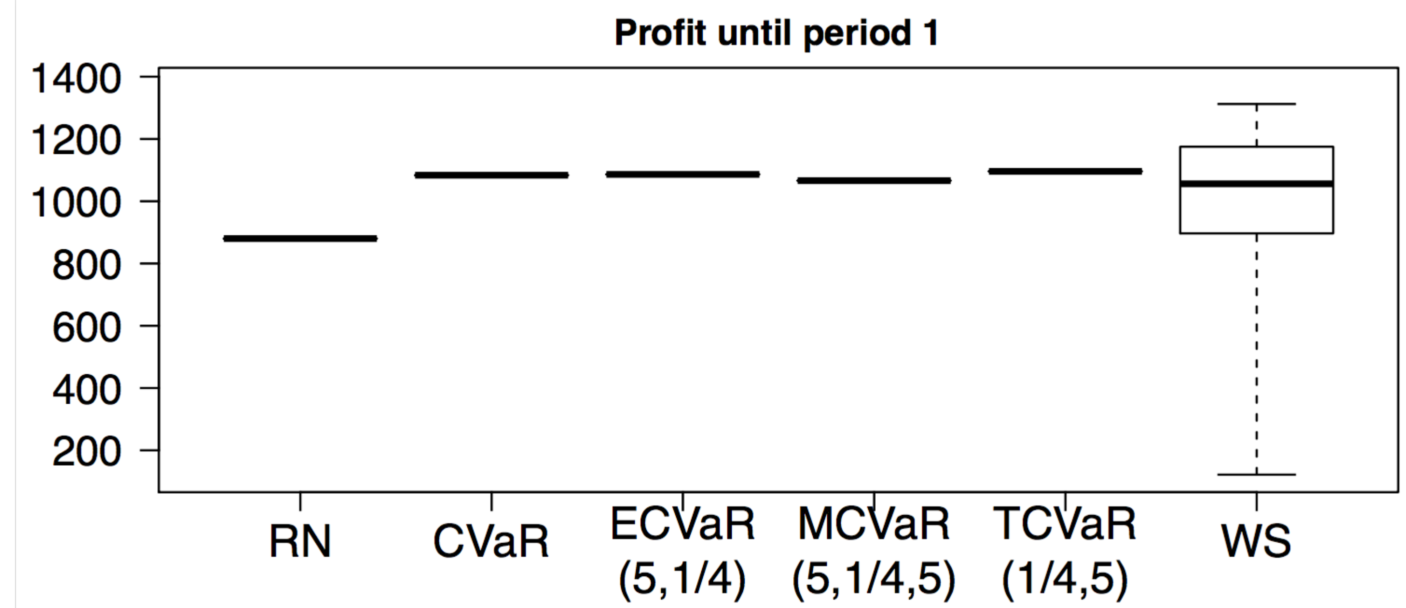
Computational results
Profit distribution

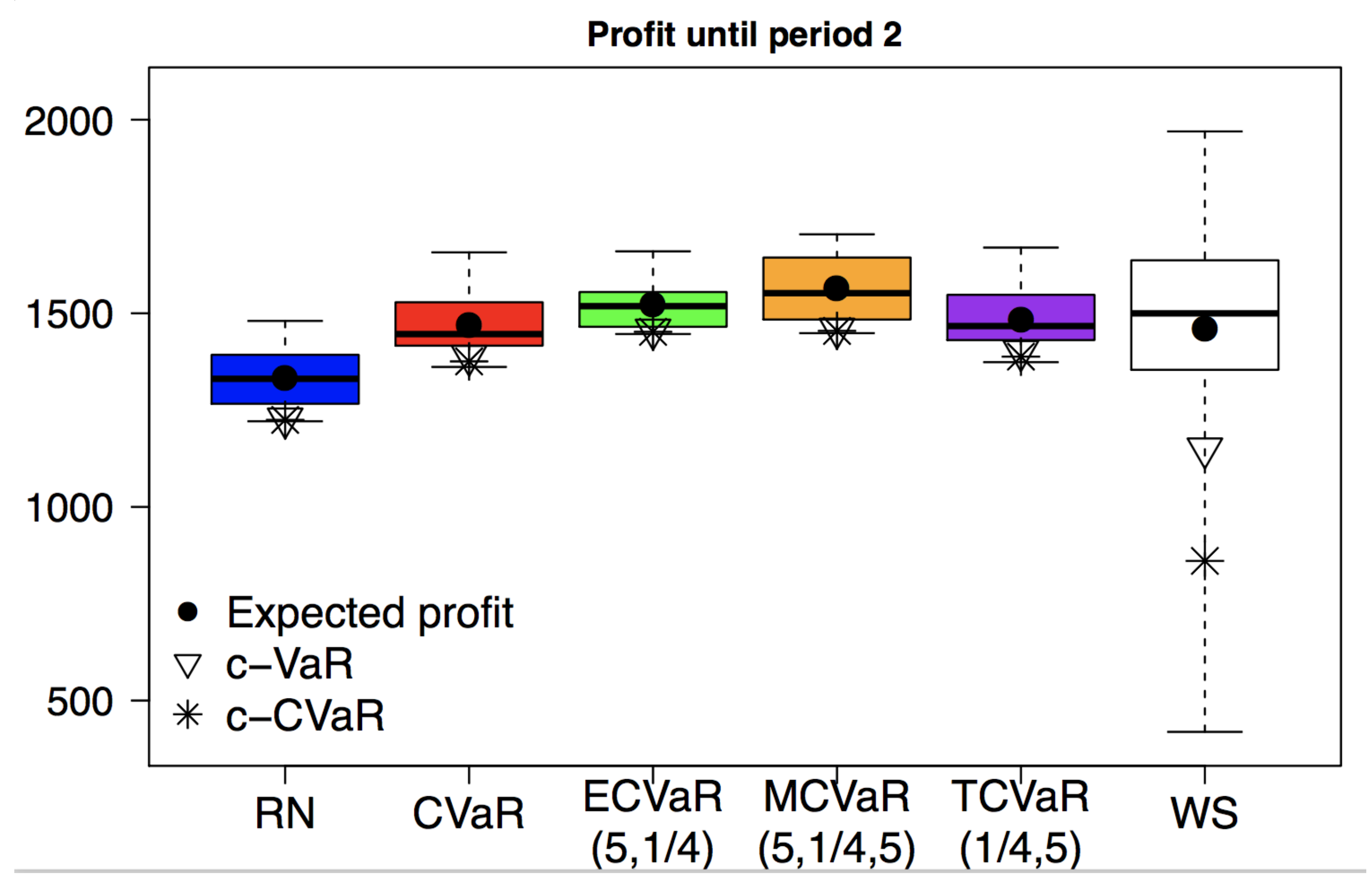




Computational results
Profit distribution







Computational results
Comparison between ECVaR, MCVaR and TCVaR

	ECVaR		MCVaR		TCVaR	
t	*#best*	*dev.best*	*#best*	*dev.best*	*#best*	*dev.best*
1	**144**	**0.00**	0	1.52	**144**	**0.00**
2	60	3.74	**72**	**1.70**	36	3.54
3	47	1.84	**54**	**1.18**	51	1.90
6	**64**	1.98	34	**1.18**	50	1.67

#best: number of scenarios for which the policy provides the best cumulative profit.

dev.best: average of the deviation from the best (in %)..

Contents

Conclusions

- An stochastic version of the forest planning problem under uncertainty in (volatile) timber prices and demand has been presented.
- Even the risk neutral approach provides a better solution than the traditional (and myopic) deterministic solution by considering the expected value of the uncertain parameters.
- Risk averse provides better results in solution's quality (since they reduce the risk of bad scenarios without reducing too much the expected profit).
- Time consistency is a desirable property for risk measures, but some other alternatives provide good practical results.

Contents

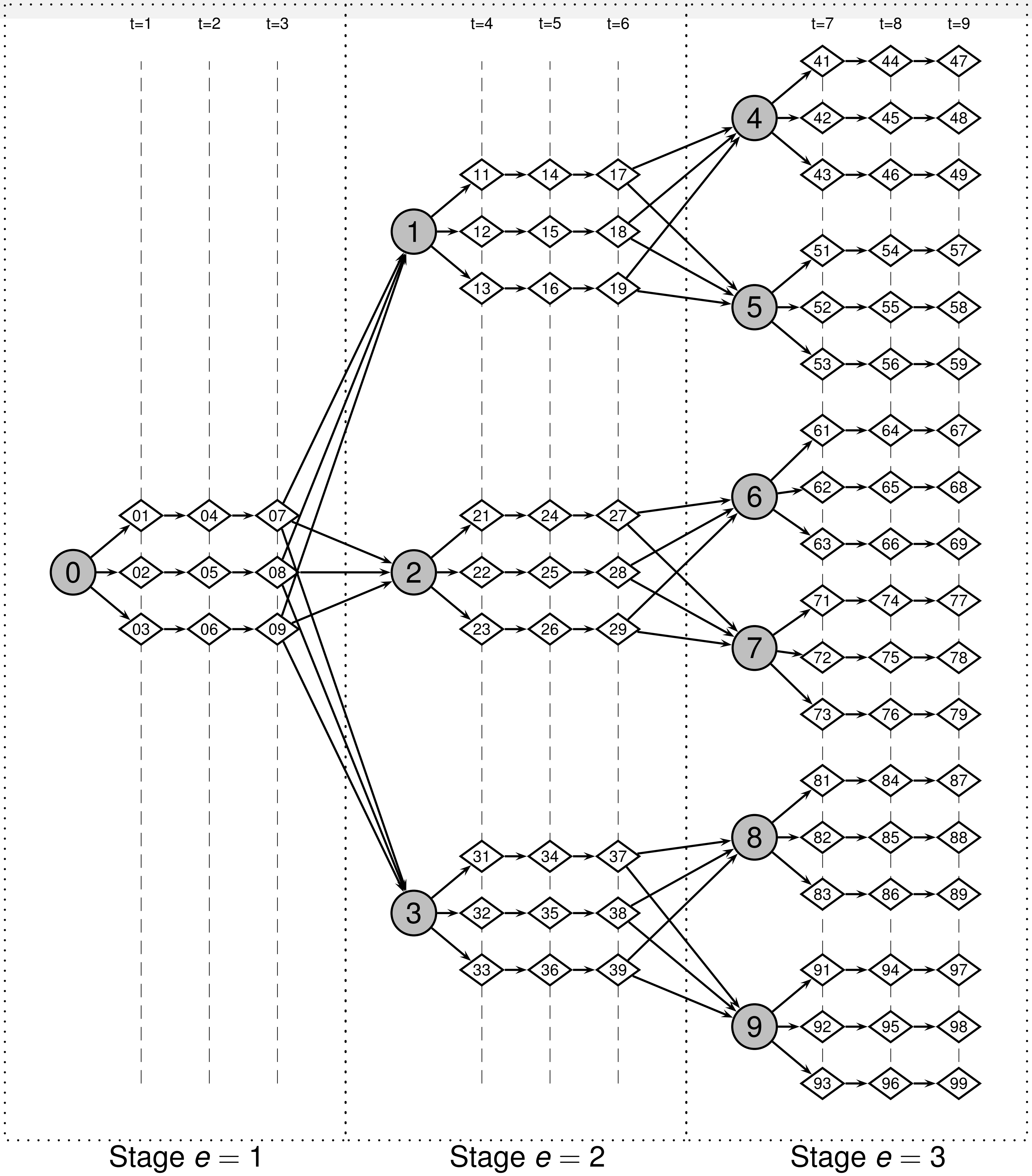
Two decision levels

Strategic (several years)

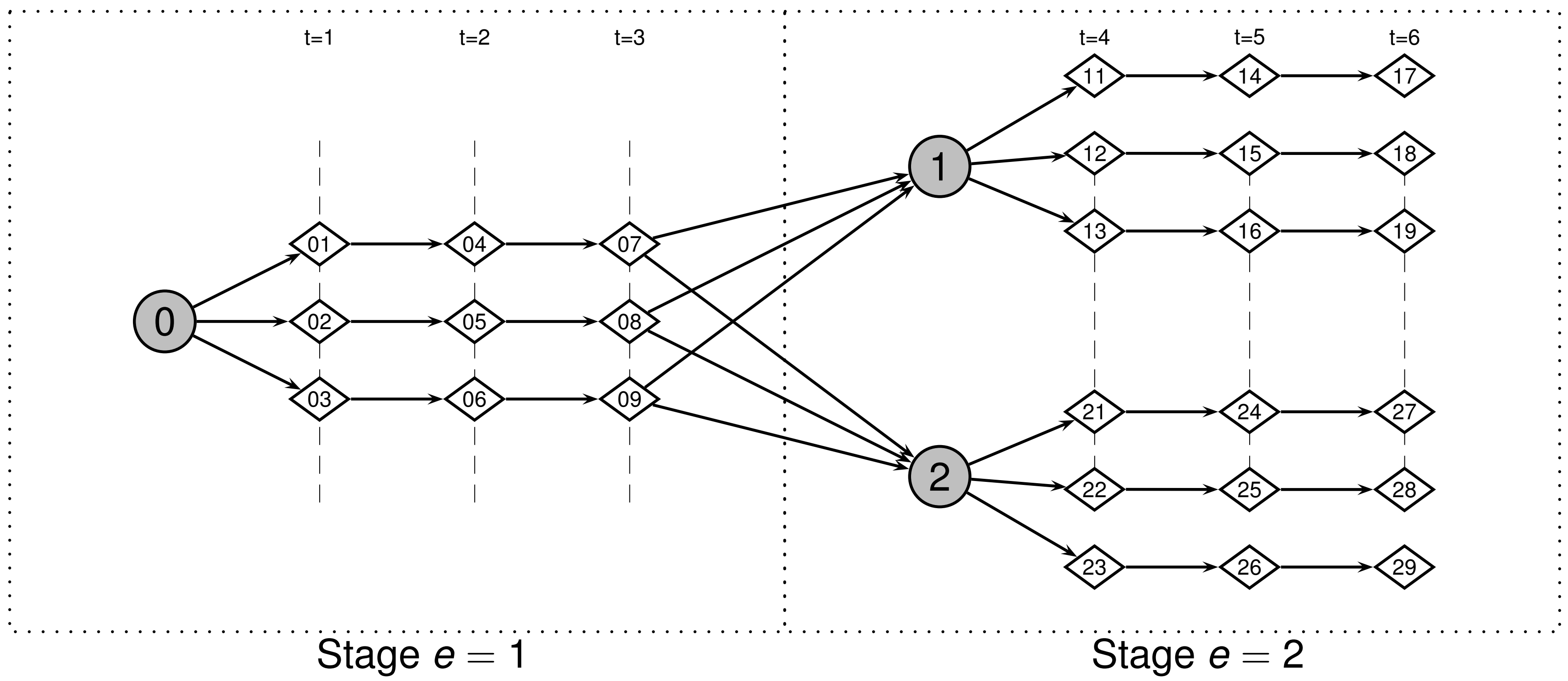
- Uncertainty: timber production.
- Quantity of timber available at each stand.
- It depends on weather conditions.
- Multistage scenario tree: stage = year.

Tactical (one year / two seasons)

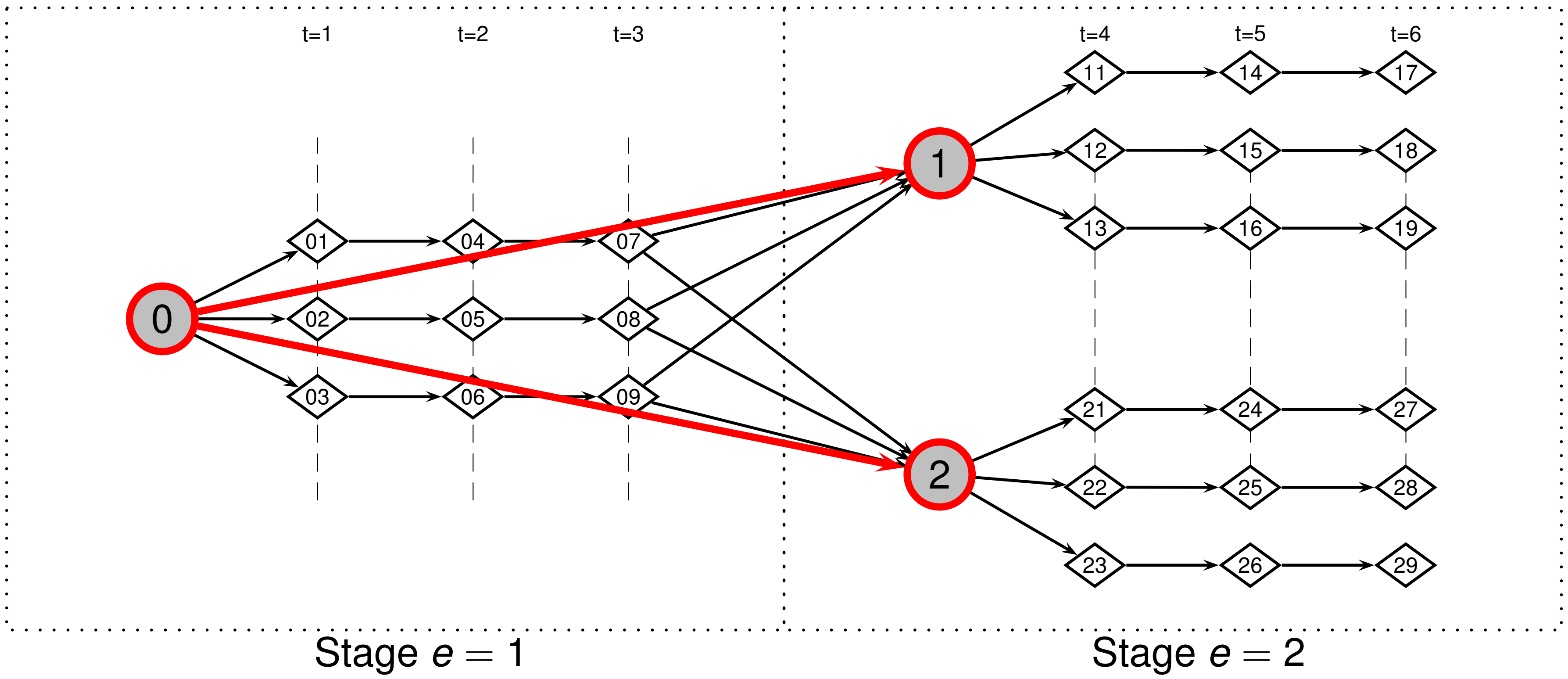
- Uncertainty: prices and demand.
- Two-stage scenario tree rooted at each strategic node.



Strategic-Tactical scenario trees

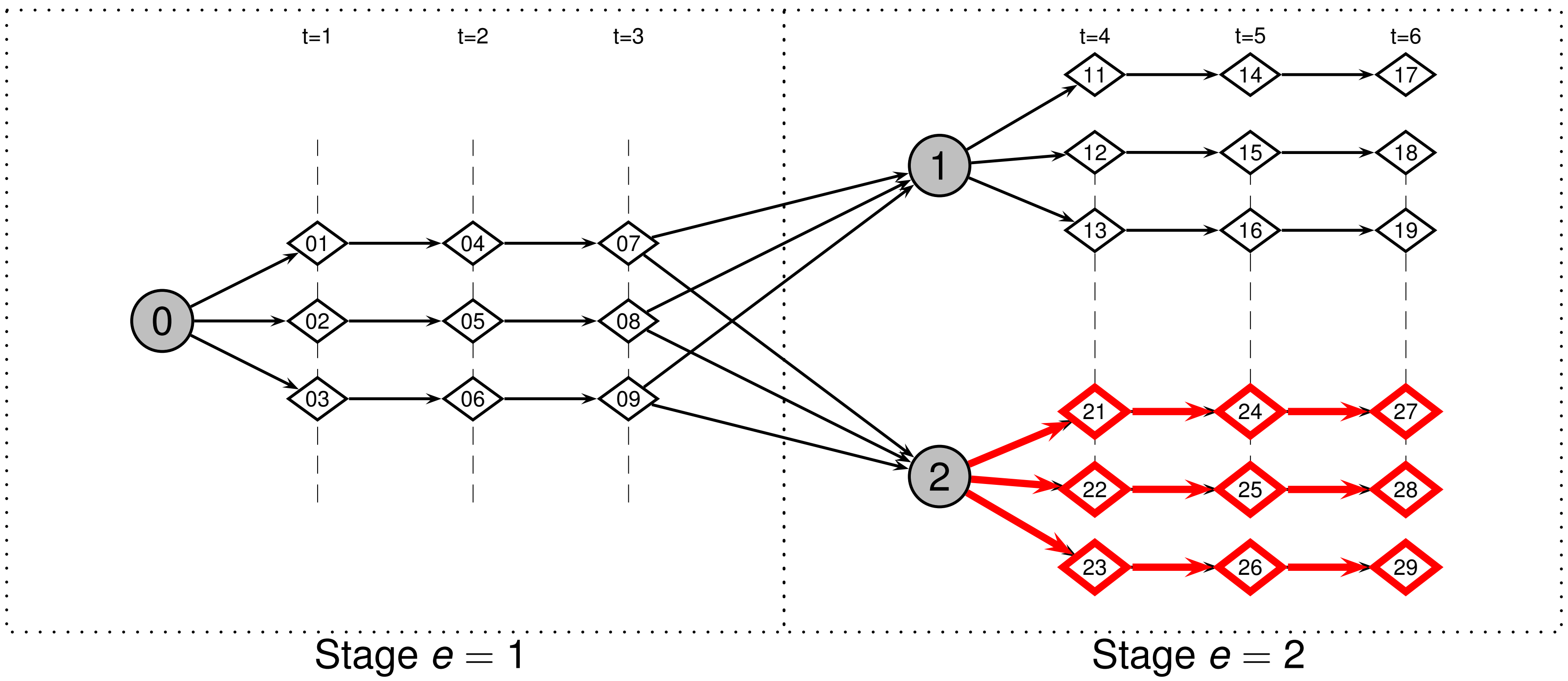


Strategic-Tactical scenario trees



Two Strategic scenarios

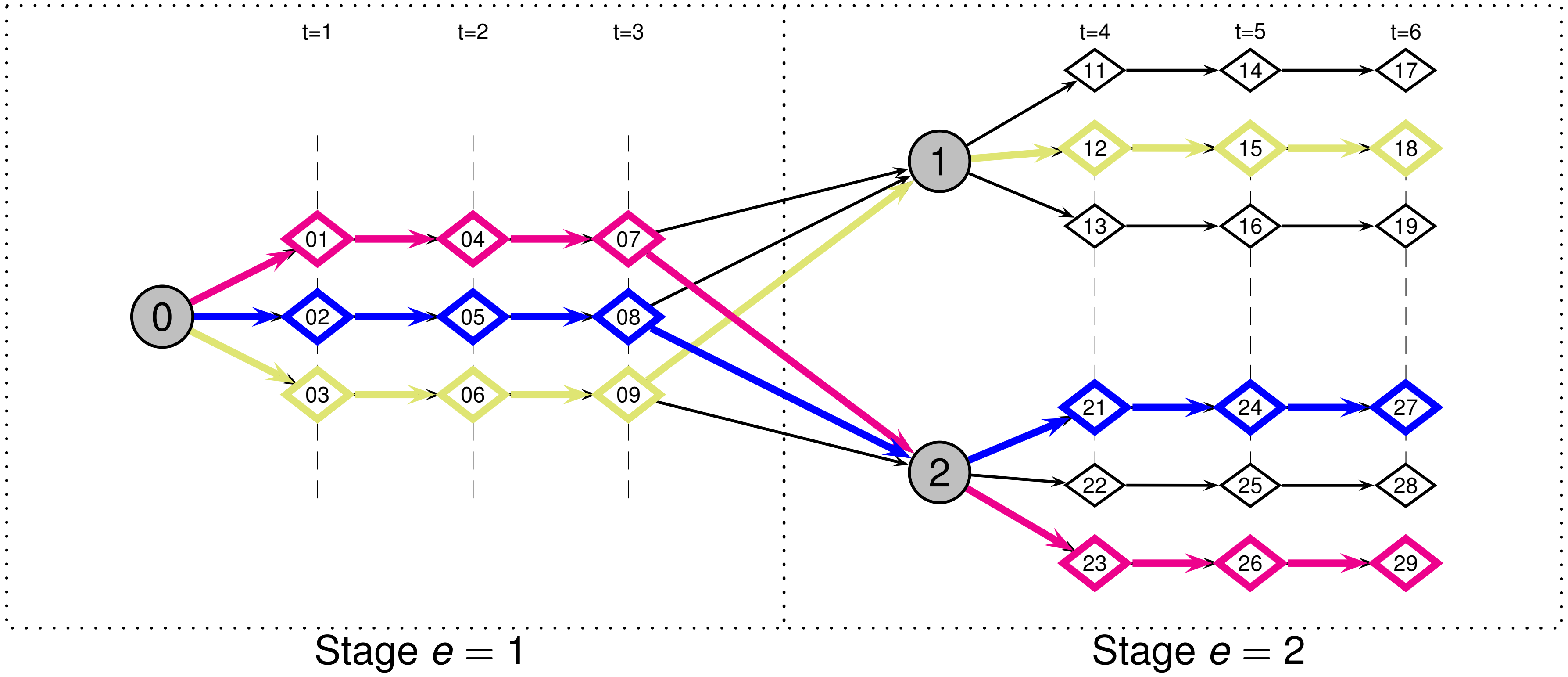
Strategic-Tactical scenario trees



Two Strategic scenarios

Three tactical scenarios rooted at each strategic node

Strategic-Tactical scenario trees



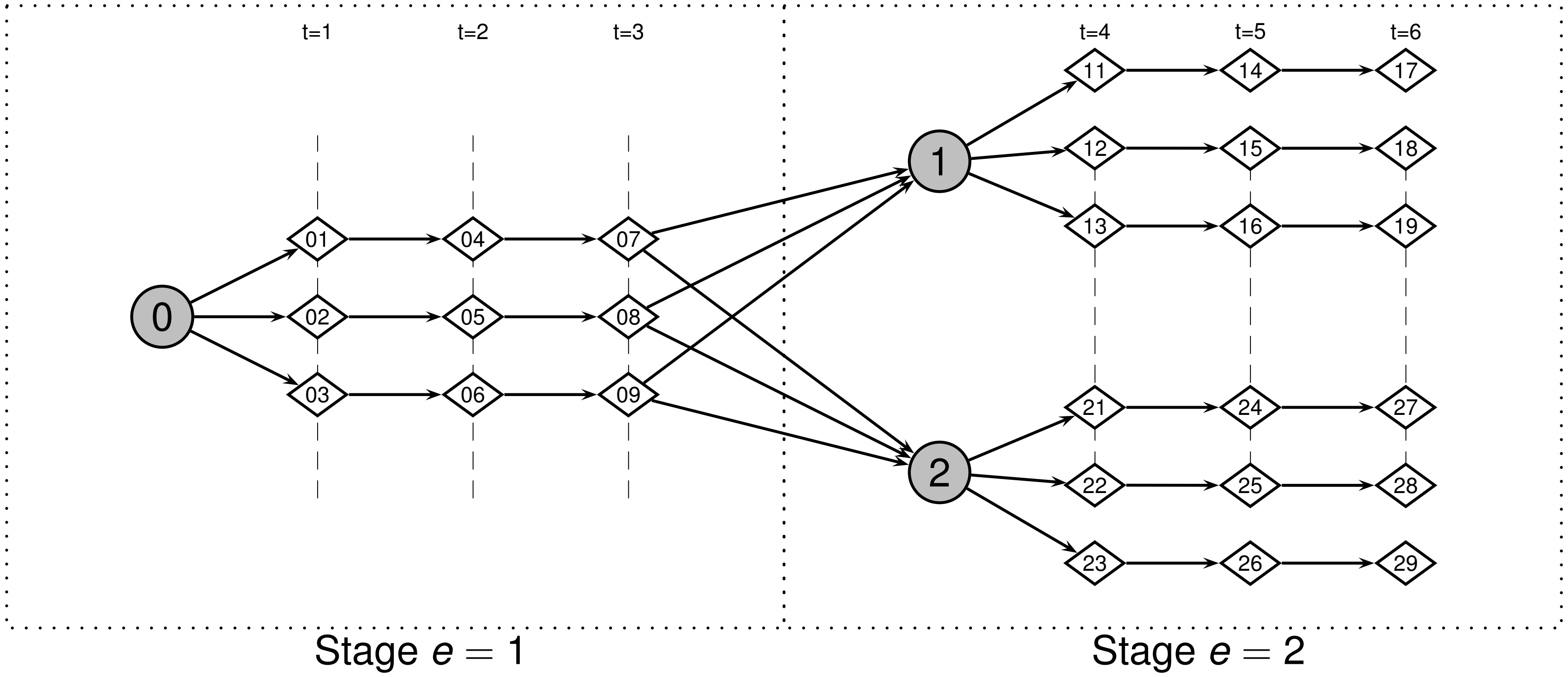
Two Strategic scenarios

Three tactical scenarios rooted at each strategic node

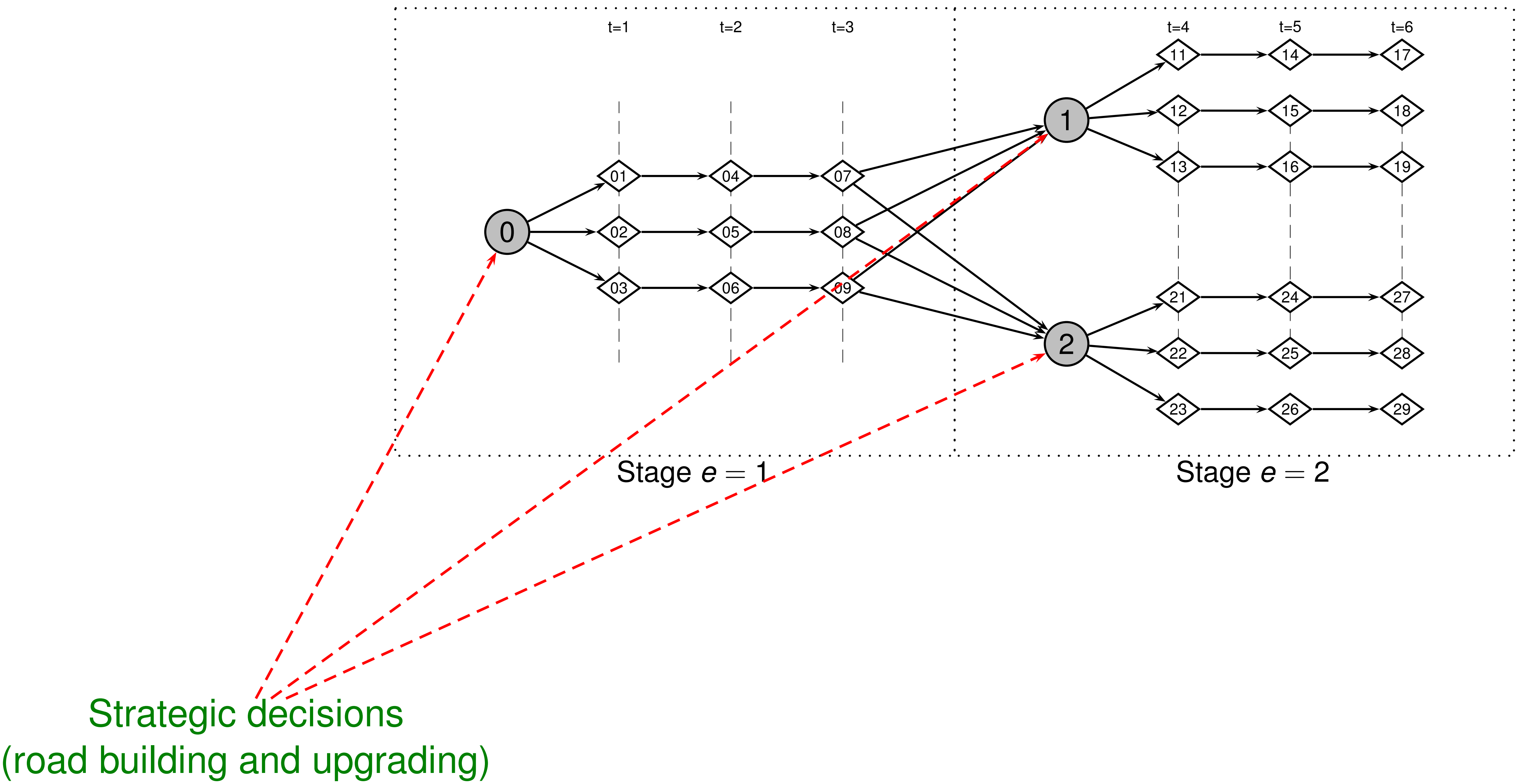
$3 \times 3 = 9$ tactical scenarios per each strategic scenario

$2 \times 9 = 18$ scenarios

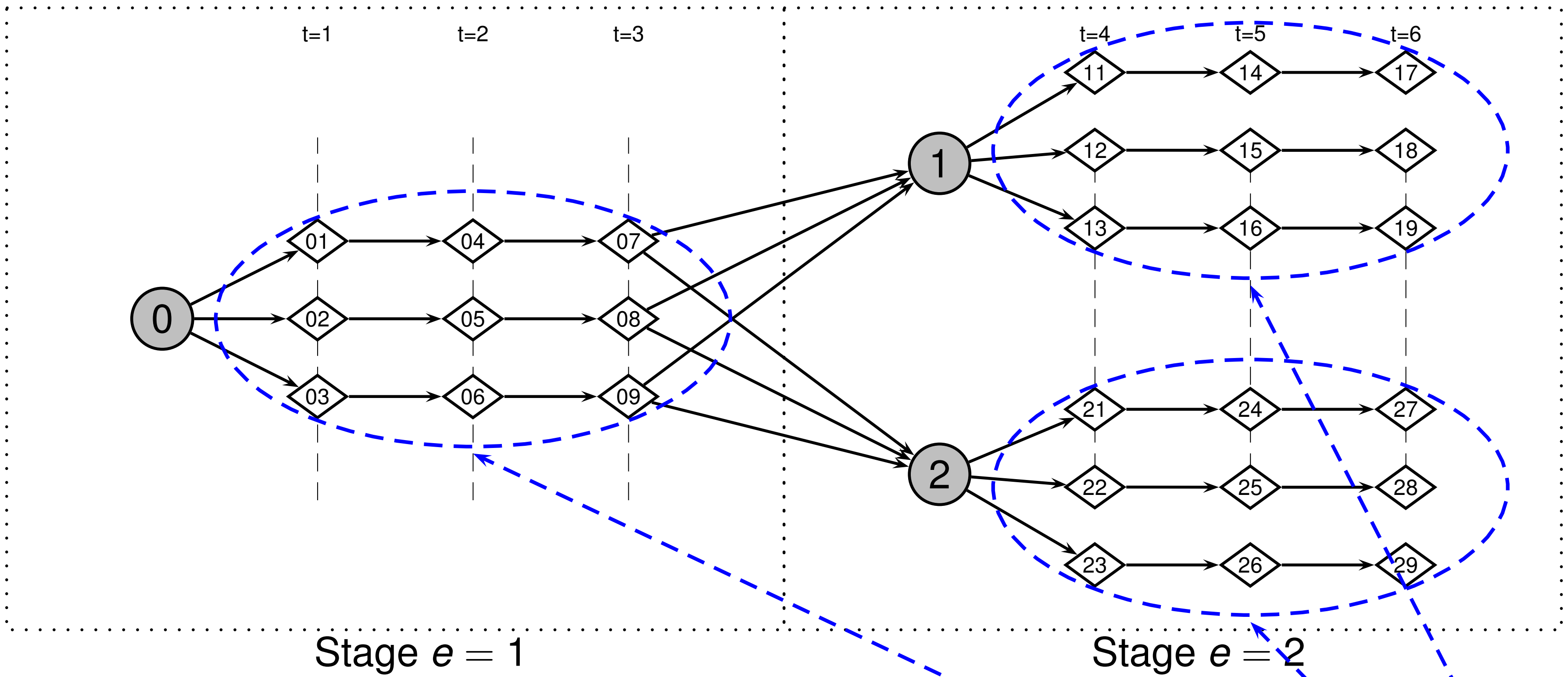
Decision levels



Decision levels

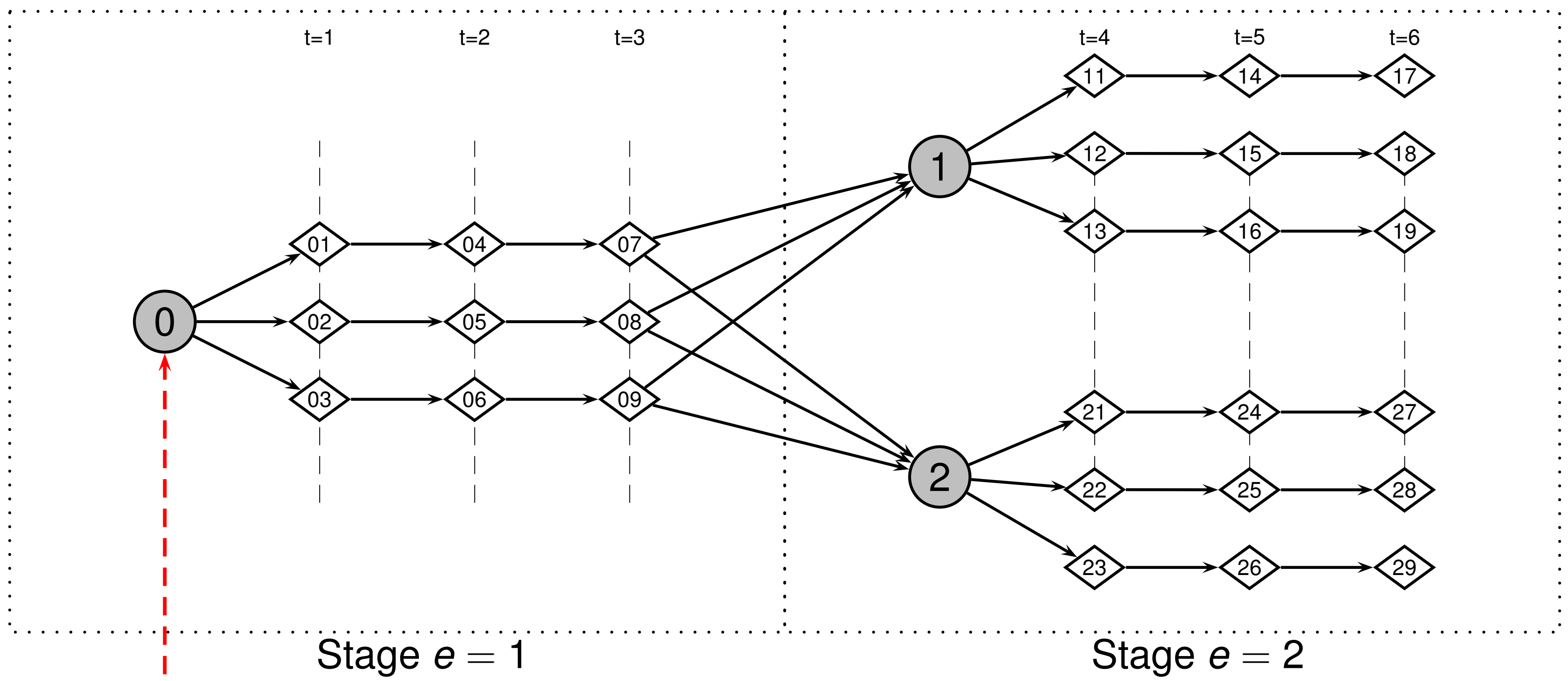


Decision levels



Tactical decisions
(timber harvesting and transport)

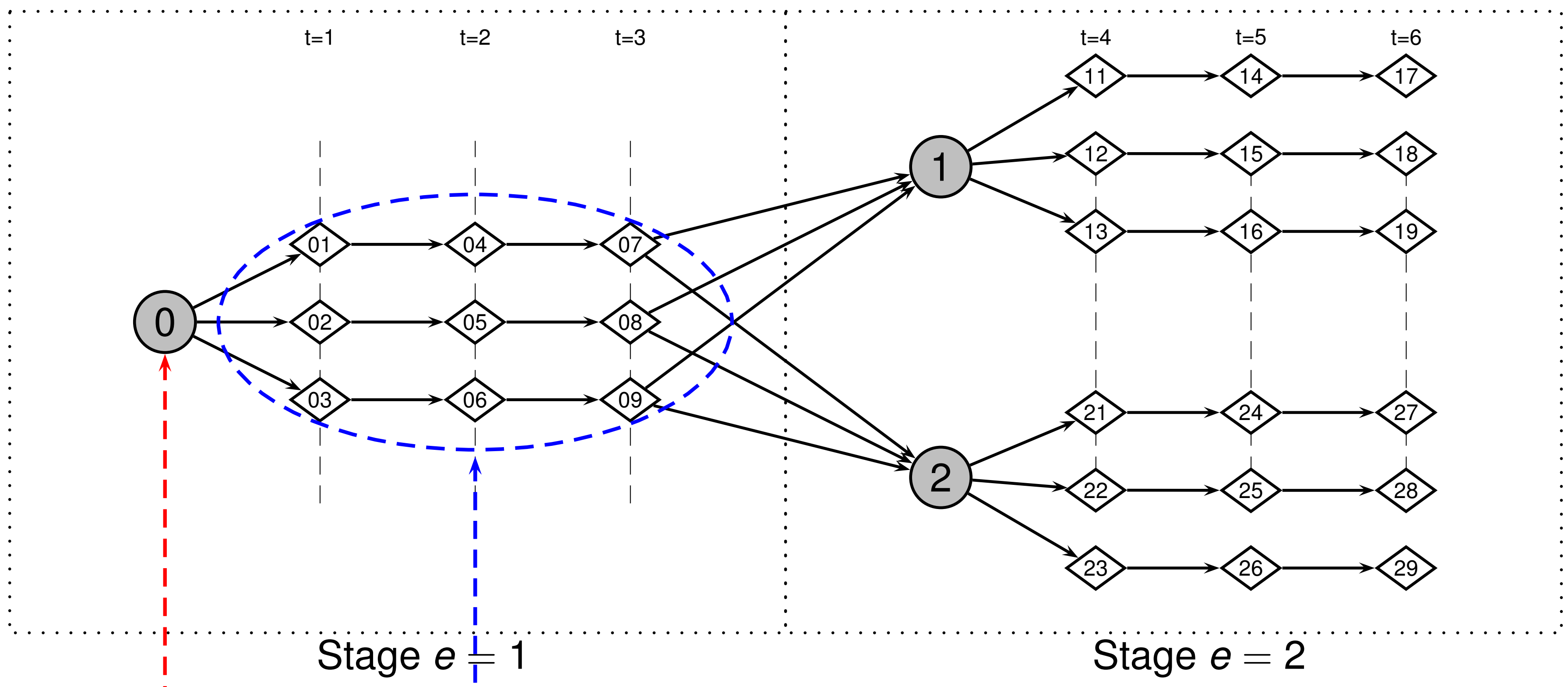
Decision levels



Strategic decisions

One node: same decisions
for subsequent scenarios

Decision levels

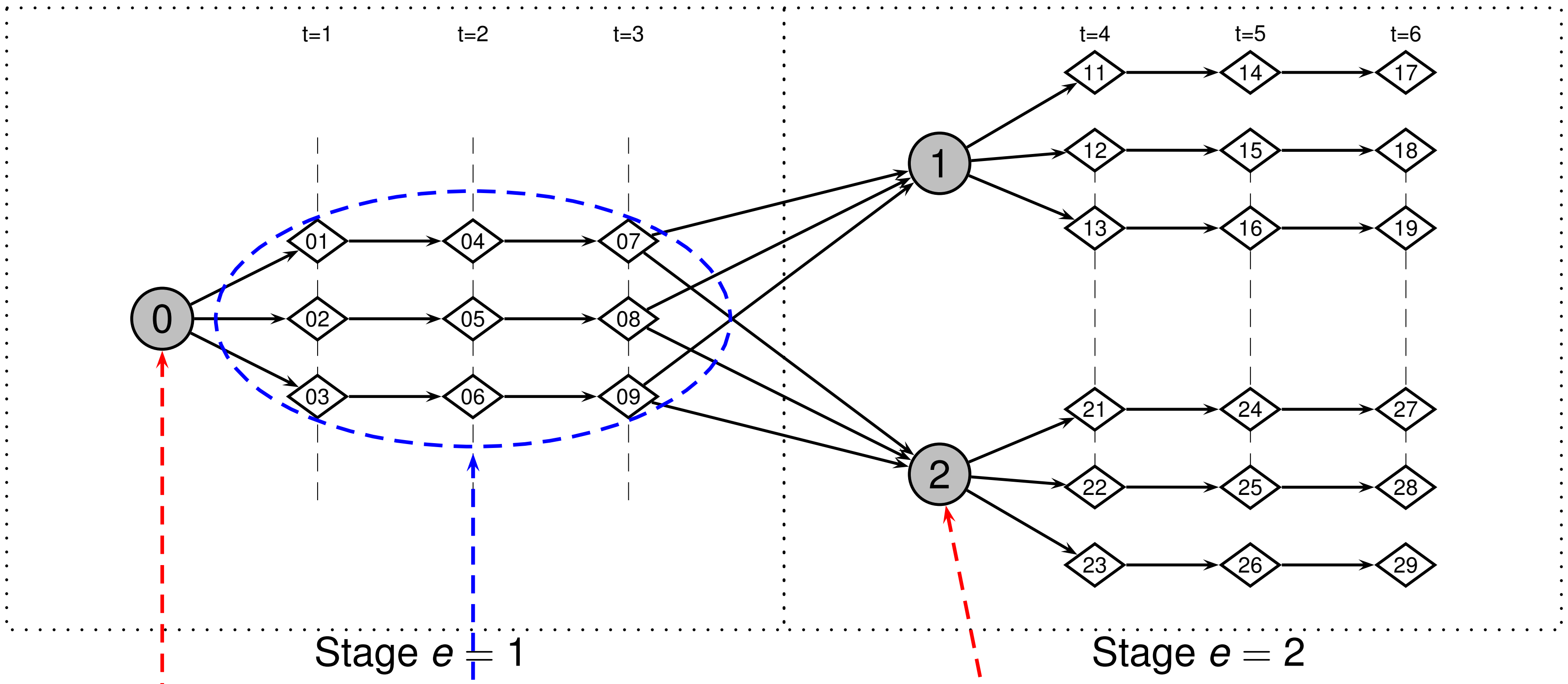


Strategic decisions

One node: same decisions
for subsequent scenarios

Tactical decisions
Several scenarios: different decisions

Decision levels



Strategic decisions

One node: same decisions for subsequent scenarios

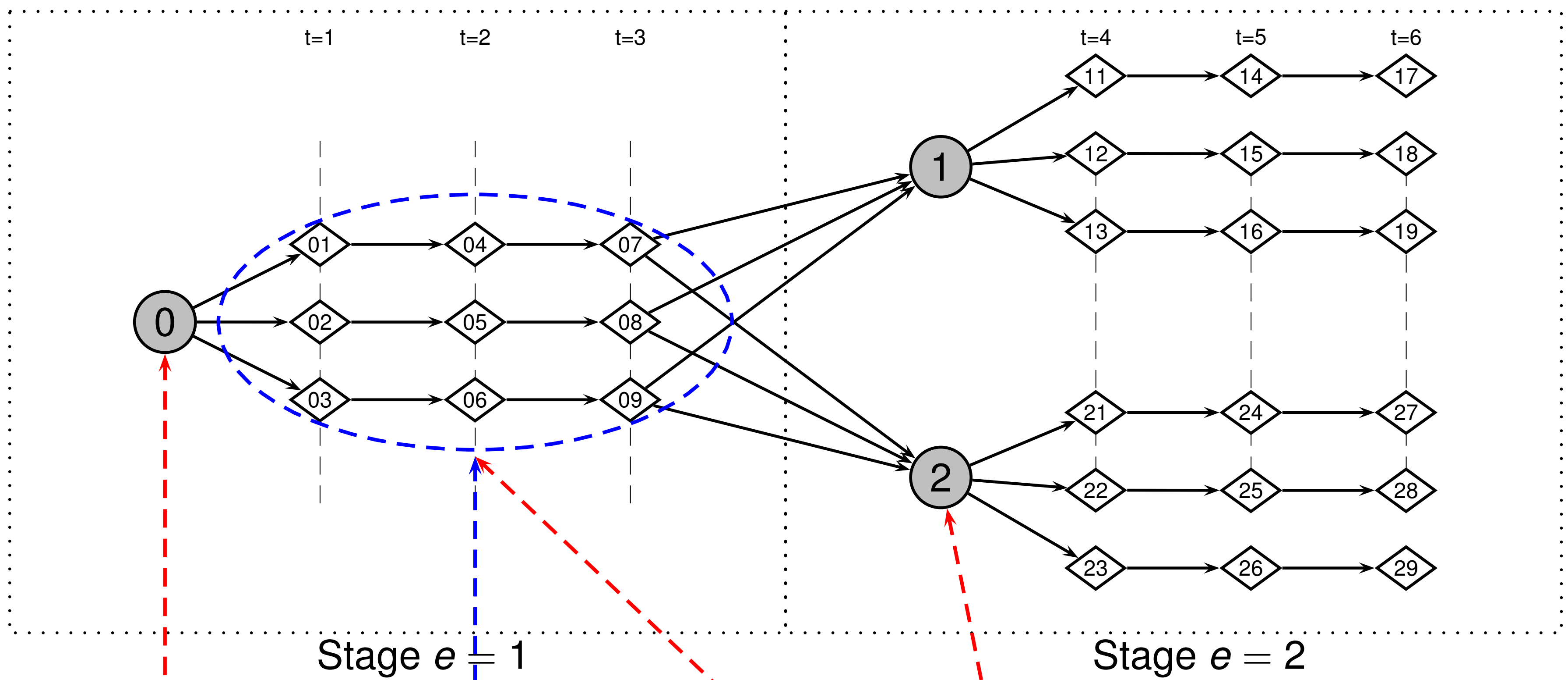
Strategic decisions

One node: same decisions for subsequent scenarios

Tactical decisions

Several scenarios: different decisions

Decision levels



Strategic decisions

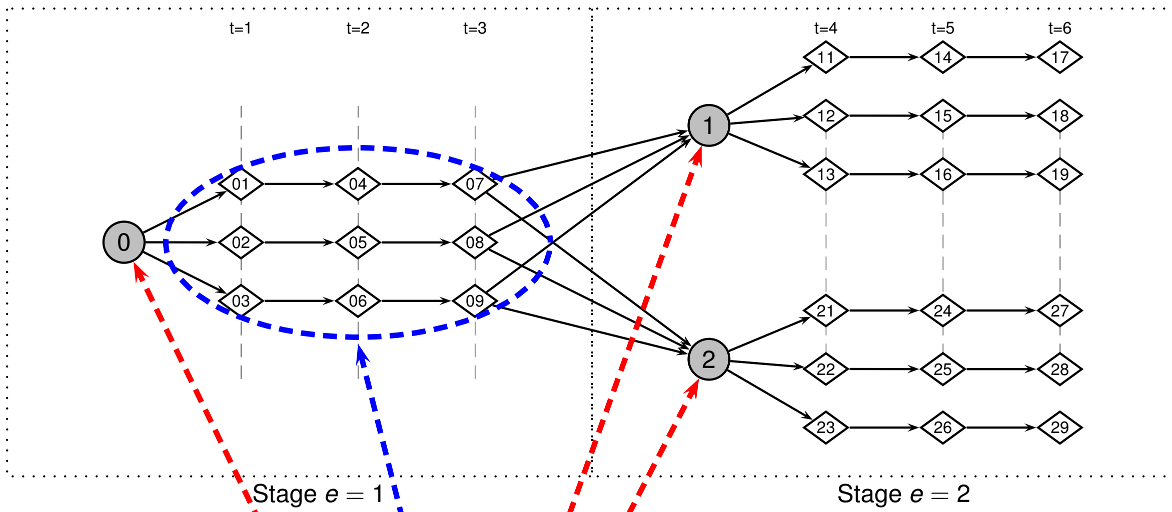
One node: same decisions
for subsequent scenarios

Strategic decisions

One node: same decisions
for subsequent scenarios
BUT ALSO FOR THE PREVIOUS ONES!!!!!)

Tactical decisions
Several scenarios: different decisions

Linking decisions between stages



Area (ha) of stand c that can be harvested, for $c \in \mathcal{C}$.

x_c^q, area (ha) of stand c that is harvested at period $t(q)$ in tactical node $q \in \mathcal{Q}$.

x_c^{*g}, maximum area (ha) of stand c harvested *by* stage $e(g)$ in the whole set of tactical nodes $\cup_{q \in \mathcal{A}_g} \mathcal{Q}_g$, for $g \in \mathcal{G}$.

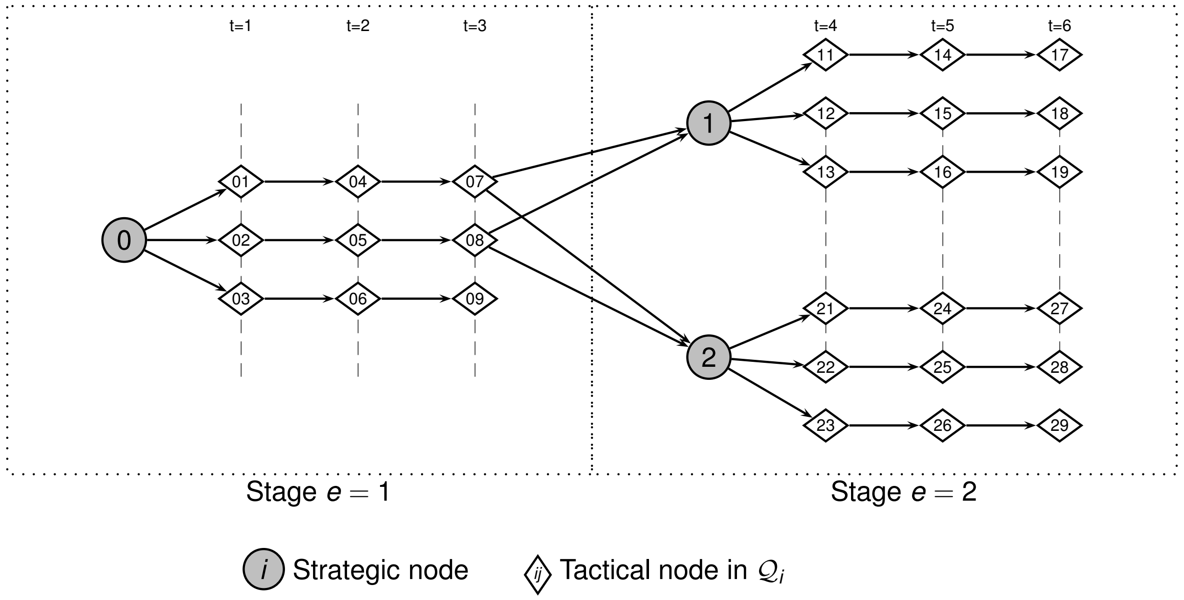
Constraints

$$\underline{A}_c e_c^q \leq x_c^q \leq \overline{A}_c e_c^q \qquad \forall c \in \mathcal{C}, q \in \mathcal{Q}_g, g \in \mathcal{G} \tag{3}$$

$$x_c^{*\sigma(g)} + \sum_{q' \in \tilde{A}_g^q} x_c^{q'} \leq x_c^{*g} \qquad \forall c \in \mathcal{C}, q \in \mathcal{L}^g, g \in \mathcal{G} \tag{4}$$

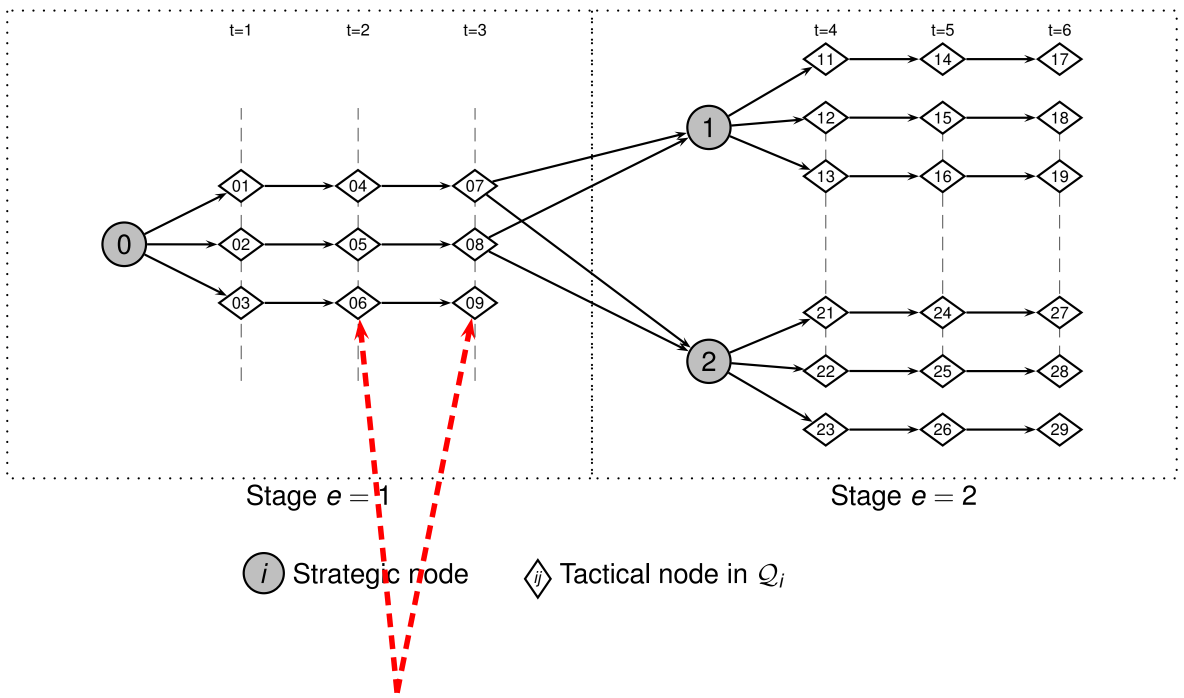
$$x_c^{*g} \leq \overline{A}_c \qquad \forall c \in \mathcal{C}, g \in \mathcal{G} \tag{5}$$

Linking decisions between stages



Timber at Stocking yards

Linking decisions between stages

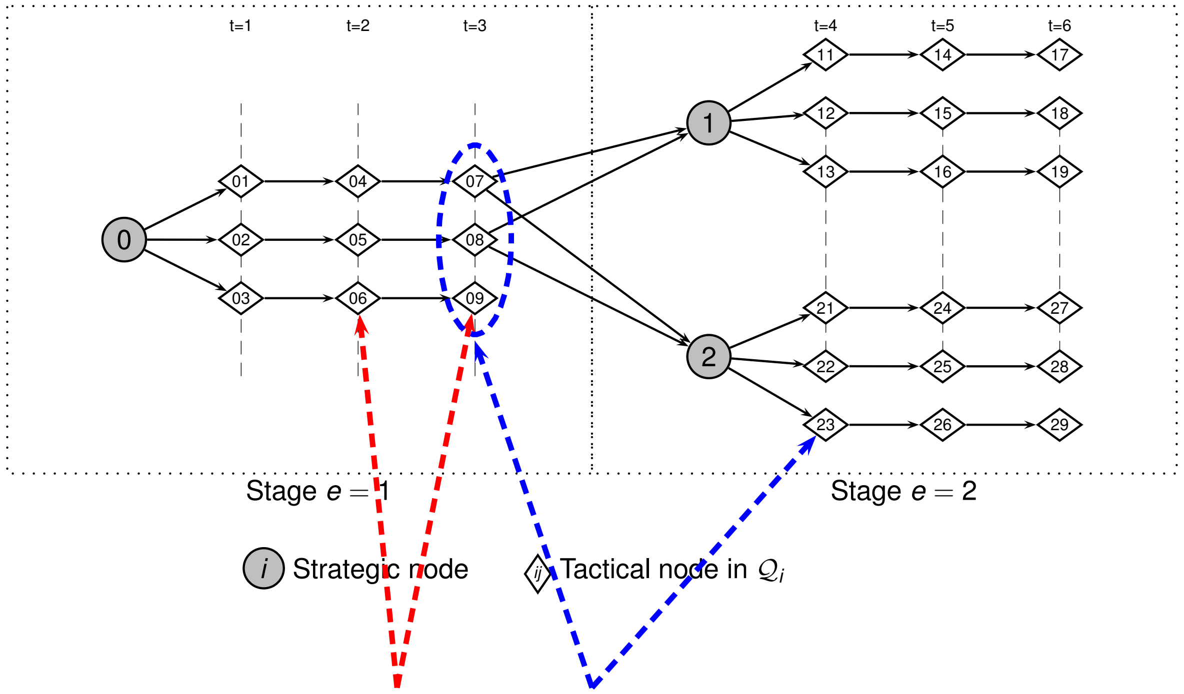


Timber at Stocking yards

- Periods belonging to the same stage:

Stock at the begining of t = Stock at the end of $t - 1$

Linking decisions between stages



Timber at Stocking yards

- Periods belonging to the same stage:

$$\text{Stock at the begining of } t = \text{Stock at the end of } t-1$$

- Periods belonging to the different stages:

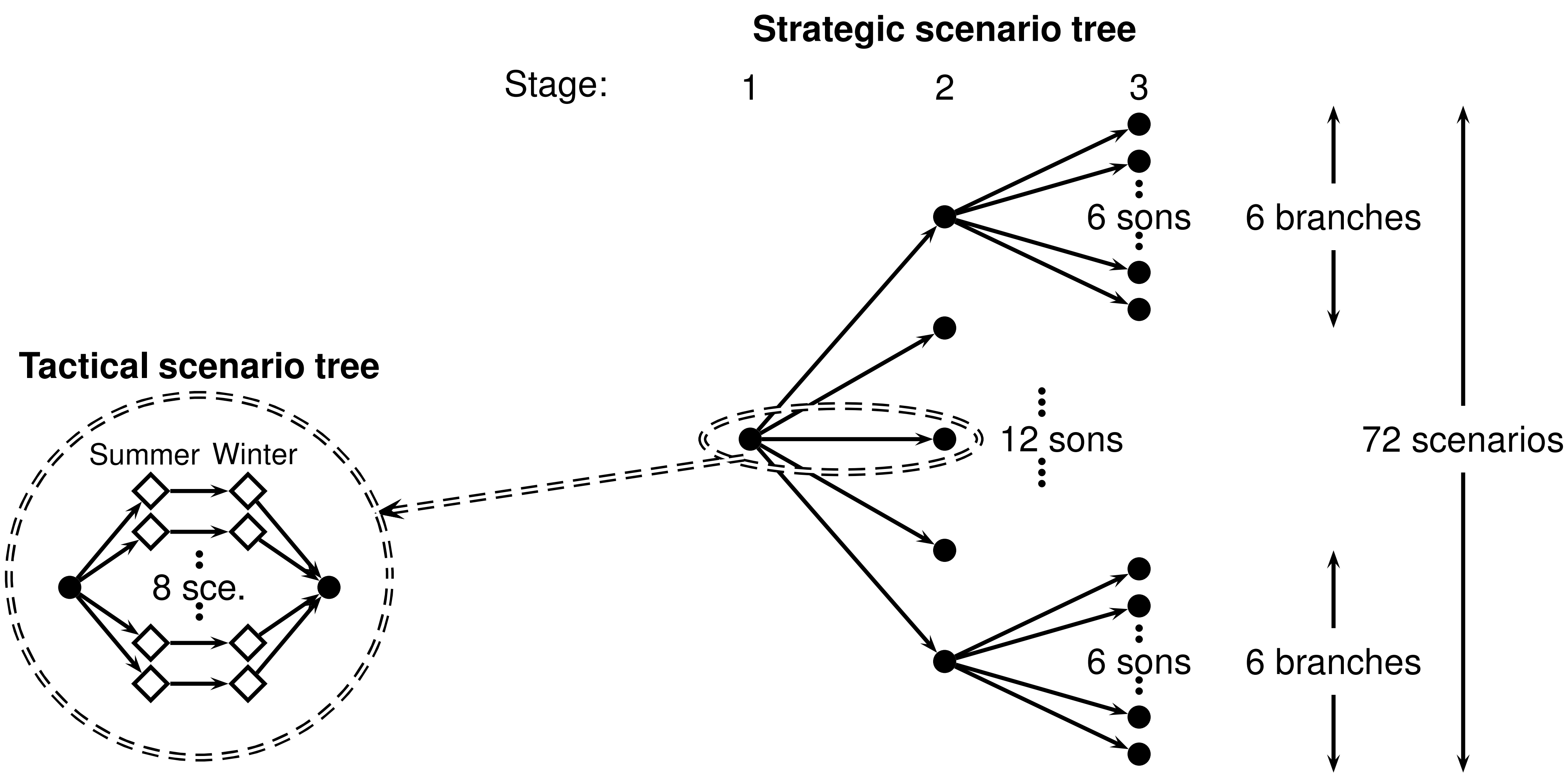
$$\text{Stock at the begining of } t = \sum_q \text{probability}_q \text{Stock at the end of } t-1$$

Contents

Case description

The forest company, Forestal Millalemu, owns 21 areas, geographically separated and connected to markets

Ins.	areas	stands	$\mathcal{I}^S$	$\mathcal{I}$	$\mathcal{R}^P$	$\mathcal{R}_1^E$	$\mathcal{R}_2^E$	Ha	$\mathcal{P}$	Markets
i1	7	29	0	43	11	23	33	989.2	3	7
i2	2	21	1	44	8	16	20	216.1	3	7
i3	1	32	1	53	4	22	25	404.1	3	7



$72 \times 8^3 =$ 36854*scenarios*!!

Models dimensions

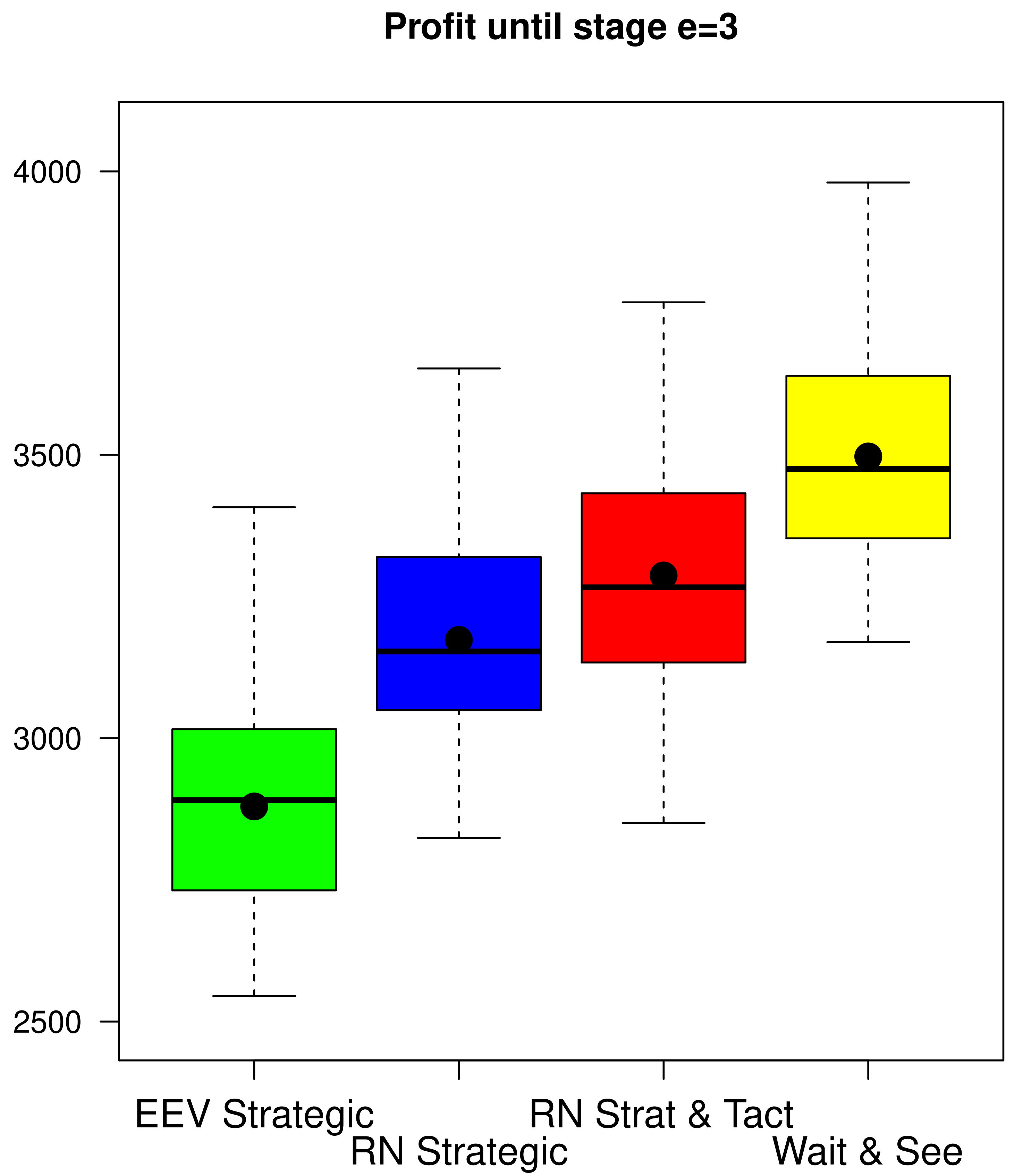
	Deterministic model			Stochastic model		
Ins.	*m*	*nc*	*n*01	*m*	*nc*	*n*01
i1	3,146	6,603	592	641,606	1,391,949	51,584
i2	2,397	4,563	427	491,372	958,109	37,307
i3	3,166	5,529	578	660,633	1,154,459	54,566

Results

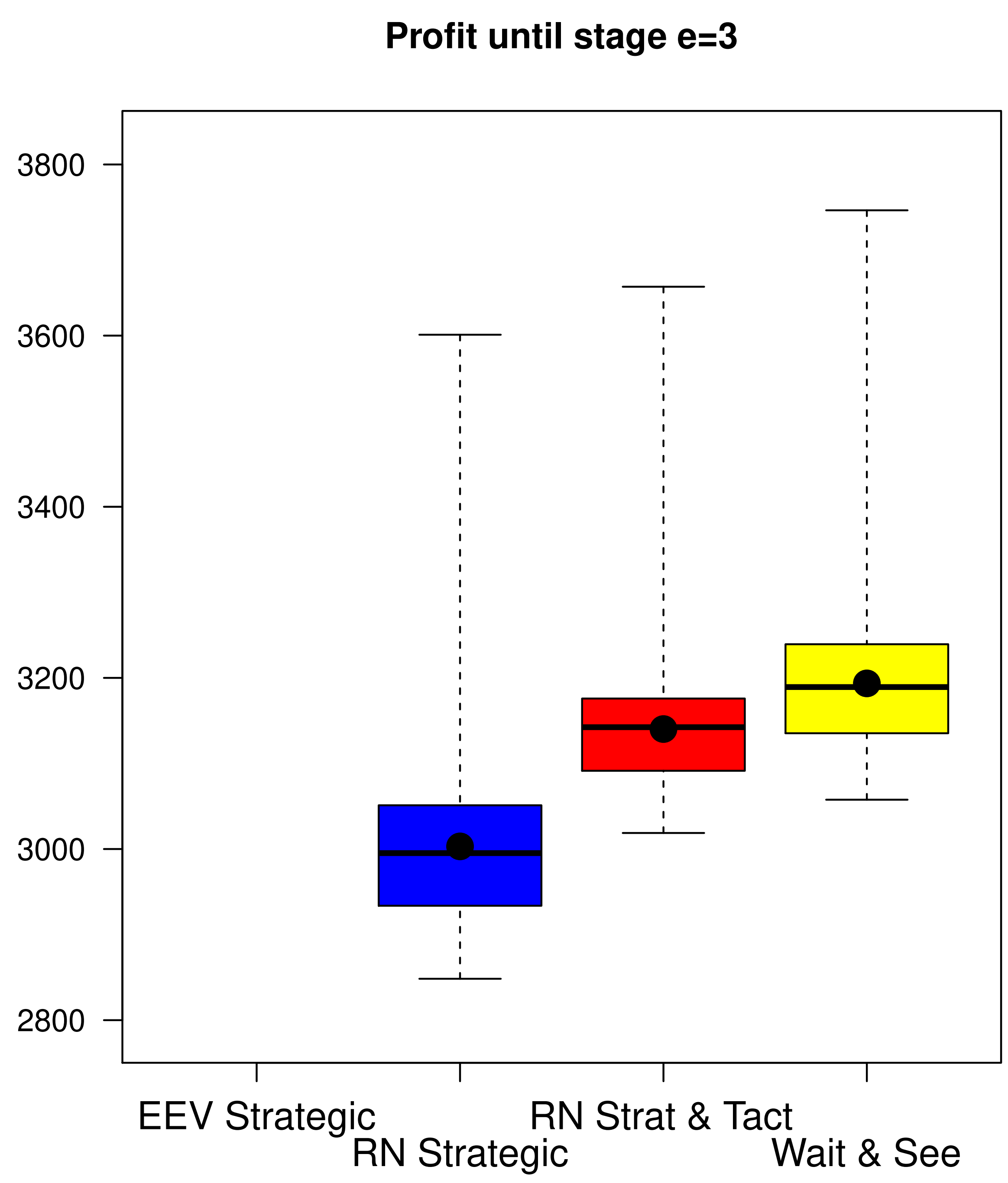
Ins.	Z_{LP}	Z_{IP}	GAP_{LP}	GAP_{OPT}	Elapsed time
i1	3374674.50	3286739.55	2.68 %	1.95 %	15 h 0 min
i2	3176521.19	3140017.68	1.16 %	0.70 %	6 h 6 min
i3	4557525.13	4501659.78	1.24 %	0.71 %	8 h 56 min

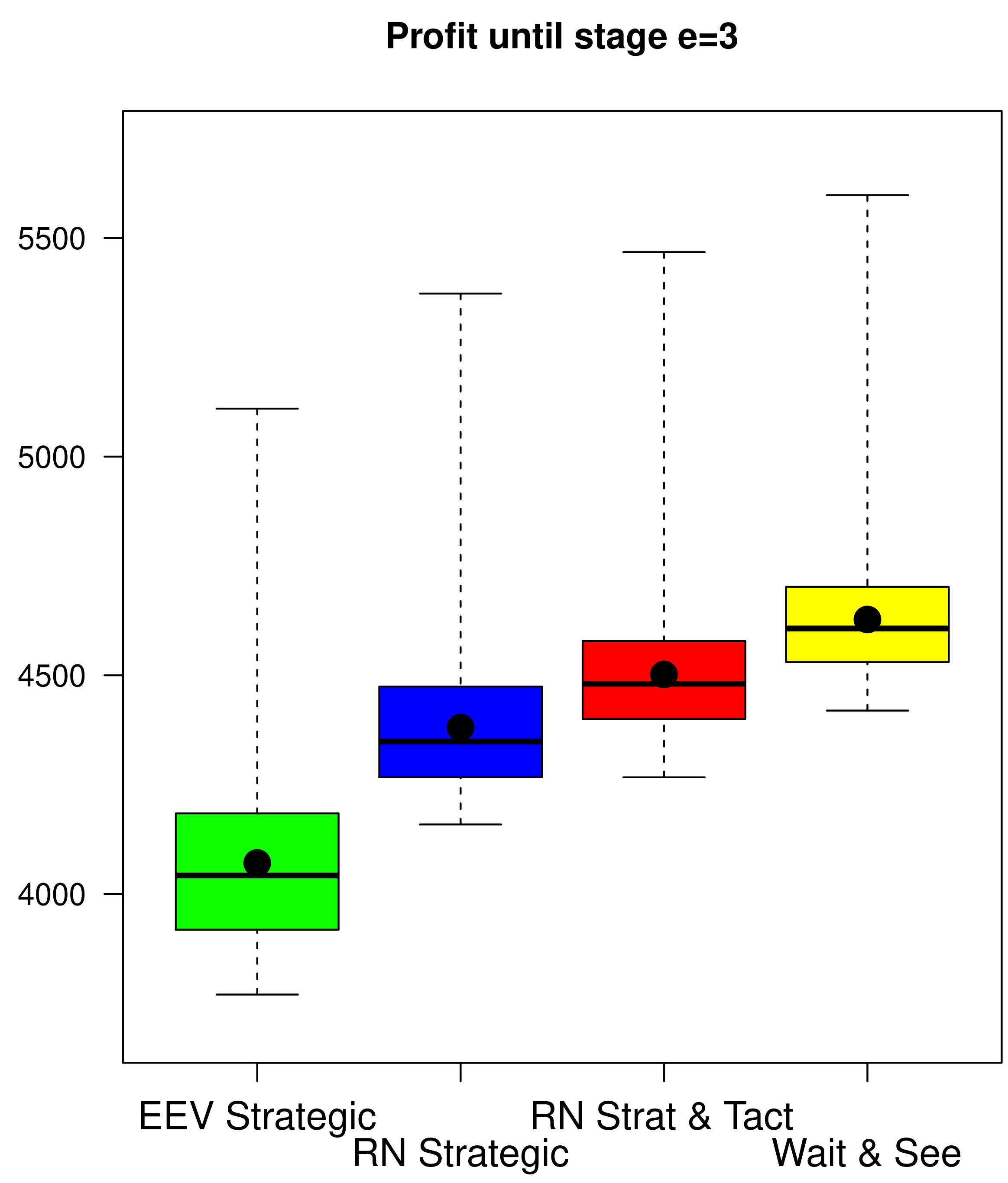
- Computer details:
  - Gams 24.2.2 and Cplex 12.6.
  - Two Intel Xeon 6 cores 2.3 Ghz, 64GRAM
- Optimality tolerance set to 1 %.
- Computing time limit: 15 hours

Results. Profit distribution (instance 1)



Results. Profit distribution (instances 2 and 3)





Contents

Conclusions

- Different levels of decisions are integrated in the same model.
- Even *small* scenario trees represent a great number of tactical scenarios.
- The proposed model provides better results in solution's quality.
- Future research: include risk aversion measures at different levels.

We invite members of ALIO and the worldwide Operations Research community to take part of XX Latin-Iberian-American Conference on Operations Research (CLAIO2020), to be held in Madrid (Spain), August 31st-September 2nd 2020. The conference is organized by the Latin-American Association of Operations Research Societies (ALIO), the Spanish Society of Statistics and Operations Research (SEIO), Universidad Complutense de Madrid (UCM) and Universidad Rey Juan Carlos (URJC). The academic program will consist of parallel, technical and special sessions, plenary talks and tutorials covering several aspects of OR.

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Conference Chairs of CLAIO 2020

Organizers

SEIO

Universidad Rey Juan Carlos

Universidad Complutense Madrid

Confirmed speakers:

Anna Nagurney. University of Massachusetts (USA)
Sebastian Ceria. Axioma (Argentina)
Emma Hart. University of Edimburg (UK)
Ángel Corberán. Universitat de Valencia (Spain)
Carlos Henggeler Antunes. Universidade de Coimbra (Portugal)

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