# Mathematical programming in an auction to provide Internet connections in Buenos Aires City schools

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# Combinatorial Auctions

- Combinatorial Auctions: Auctions where the bids consist of groups of items, thus turning the decision process into a combinatorial optimization problem.

- These models comprise many aspects, including the auction design, the development of mathematical models for determining the auction outcome, and the implementation of suitable algorithms for solving these models.

- The organizer seeks to minimize its costs if it is a "buyer", or to maximize its benefit if it is a "provider".

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- A book surveying the state of the art is *"Combinatorial Auctions"*, Peter Cramton, Yoav Shoham, and Richard Steinberg (2006), MIT Press.

- A successful case:
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- The National Bureau of Help and Grants for Schools (JUNAEB) in Chile provides meals to around ten thousand schools in Chile.

- Different kinds of meals are supplied for more than two million children at school ages.

- The chilean government spends approx. 500 million dolars per year in this program.

- The meal supply is performed by catering companies, which take part in annual auctions organized by JUNAEB.

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- For practical purposes, each of the 13 regions (provinces) in Chile is subdivided in Territorial Units (TU). There are over 100 TUs.

- Each company taking part in the auction presents bids for groups of TUs. Each bid for a group of TUs is either accepted or not, thus taking advantage of economies of scale.

- Each year one third of the country is auctioned, and the contracts last three years.

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- Since 1997, a mathematical model developed by the University of Chile has been regularly applied, in order to support the decision-making process. The model objective consists in finding the set of bids covering all the auctioned TUs and minimizing the total expenses.

- Thanks to the implementation of this model, savings of over 40 million dollars per year have been attained.

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# Our problem

Auction the Internet connection service for the 709 school buildings in Buenos Aires City, among the interested Internet providers.
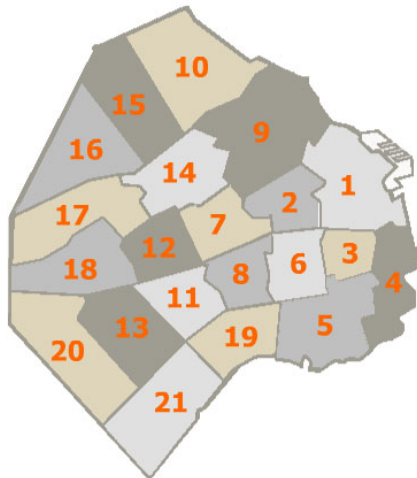
# Original City Government proposal

- Auction each school separatedly.
- Each Internet provider makes a bid for each school.
- Each school is assigned the Internet provider offering the best price for that school.
- Problems: no economies of scale captured; possibility of collusion; possible price distortion (higher prices for schools in areas with little competence).

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First attempts
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Integer programming model

# Our first proposal

- Basic units: the School Districts.
- The companies bid for groups of districts (as in the JUNAEB auction).
- Main problem: the coverage areas of the companies do not conform to the borders of the School Districts (it is much more expensive to install the technology outside the actual coverage area), and bids would be affected.

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# School Districts

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# Coverage areas of the potential bidders

Cablevision
=
Multicanal

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# Coverage areas of the potential bidders

Telefónica

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# Coverage areas of the potential bidders

Telecom

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# Coverage areas of the potential bidders

Telmex

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# Coverage areas of the potential bidders

iPlan

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# Coverage areas of the potential bidders

Metrotel

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# Our second proposal

- Design new territorial units taking into account the coverage areas.



- Problems:
  1. some coverage areas go across these new TUs,
  2. possibility of collusion in the TUs with only two competing companies.

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# Our third proposal: the chosen model

- Each company provides a price for a single school (the same for all schools).
- Each company provides a list of schools it can serve.
- Each company provides volume discounts (with prespecified price intervals).
- Upper bound on the number of schools each company can receive (proposal finally not accepted).

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# Our third proposal: the chosen model

- Benefits:
    1. Almost impossible to collude.
    2. A company cannot impose a high price in areas with low competence, and propose lower prices in areas with higher competence.
    3. Economies of scale are captured.
    4. The company itself defines its coverage area through the list of schools it can serve.

- Observation: Note that this is a "multi-unit auction". We call this auction "logistic multi-unit", whose principal characteristic is the presence of strong supplier-cost heterogeneity due to logistical factors. This characteristic results in cost advantages or disadvantages for some potential bidders relative to others on the units being tendered.

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# Foundations of the mathematical model

- The optimal solution for the city is obtained, assigning an Internet provider to each school.

- The territorial units are designed after the bids are presented, in terms of the lists of schools provided by the companies (note that this formulation is more efficient than the natural formulation with one binary variable per school and company, which has much more variables).

- The model solution determines how many (and not which) schools each company receives at each territorial unit (each unit it not neccessarily assigned as a whole).

- After the model is solved, if there are territorial units shared by two or more companies, a manual or automatic process can be applied to assign the schools to the companies (for example, by following geographically based criteria).

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# The integer programming model

Model parameters:

- $C$: set of Internet providers/companies
- $R$: set of territorial units/regions, defined by the intersection of the companies' coverage areas
- $E_r, \ r \in R$: set of schools in the region $r$
- $p_{ji}, \ j \in R, \ i \in C$: 1 if the company $i$ bids in the region $j$, 0 otherwise
- $T$: set of price intervals (0–19, 20–39, ..., 80–99, 100–149, 150–199, 200–299, ..., 500–600, 600–709)
- mín($t$) and máx($t$), $t \in T$, lower and upper bounds of each price interval
- $c_{ti}, \ t \in T, \ i \in C$: cost per school in the price interval $t$ offered by the company $i$

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# The integer programming model

Model variables:

- $x_{ji} \in \mathbb{Z}_{\geq 0}$, $j \in R$, $i \in C$: number of schools in the region $j$ assigned to the company $i$

- $y_{it} \in \{0, 1\}$, $i \in C$, $t \in T$: specifies whether the company $i$ is using the price interval $t$

- $z_{it} \in \mathbb{Z}_{\geq 0}$, $i \in C$, $t \in T$: number of schools assigned to the company $i$ in the price interval $t$. It can be defined in $\mathbb{R}_{\geq 0}$, since in the optimal solution this variable will be set at an integer value.

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# The integer programming model

$$\min \sum_{i \in C} \sum_{t \in T} c_{ti} z_{it}$$

$$\sum_{i \in C} x_{ji} = |E_j| \qquad \forall j \in R$$

$$\sum_{j \in R} x_{ji} \geq \min(t) - M(1 - y_{it}) \qquad \forall i \in C, \ \forall \ t \in T$$

$$\sum_{j \in R} x_{ji} \leq \max(t) + M(1 - y_{it}) \qquad \forall i \in C, \ \forall \ t \in T$$

$$\sum_{t \in T} y_{it} = 1 \qquad \forall \ i \in C$$

$$z_{it} \geq \left(\sum_{j \in R} x_{ji}\right) - M(1 - y_{it}) \qquad \forall \ i \in C, \ \forall \ t \in T$$

$$x_{ji} = 0 \qquad \forall \ i \in C, \ \forall \ j \in R \ /p_{ji} = 0$$

$$\sum_{t \in T} z_{it} = \sum_{j \in R} x_{ji} \qquad \forall \ i \in C$$

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# Searching for multiple optima

- Let $x_{ij} = a_{ij}$ be the optimal solution, for every company $i$ and region $j$.
- For each such equation, we add two binary variables $w_{ij}$ and $w'_{ij}$. Furthermore, we add the following constraints:

$$
\begin{aligned}
x_{ij} &\geq (a_{ij} + 1)w_{ij} & \forall ij : a_{ij} \neq 0 \\
709 - x_{ij} &\geq (709 - (a_{ij} - 1))w'_{ij} & \forall ij : a_{ij} \neq 0 \\
\sum_{a_{ij} \neq 0} (w_{ij} + w'_{ij}) &= 1
\end{aligned}
$$

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# Distribution within a competition unit

- Once the model has been solved and the final solution has been obtained, schools must be assigned to the bidders in each region. This step must be performed in a post-processing phase since the model does not assign individual schools to bidders, but specifies how many schools are to be assigned to each bidder at each region instead.

- A straightforward greedy procedure consists in panning the region from north to south, and assign to the bidders the required number of schools.

- This way, the "sub-regions" assigned to each bidder tend to be compact areas, favoring the logistics of each company. If the region is longer in the east-west axis than in the north-south axis, then we can consider a west-to-east panning.

# Received bids

Cablevisión
709 schools
(whole set of
schools)
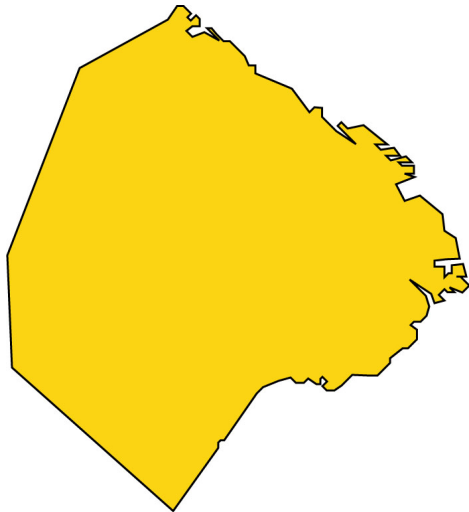
# Received bids



Telefónica
348 schools
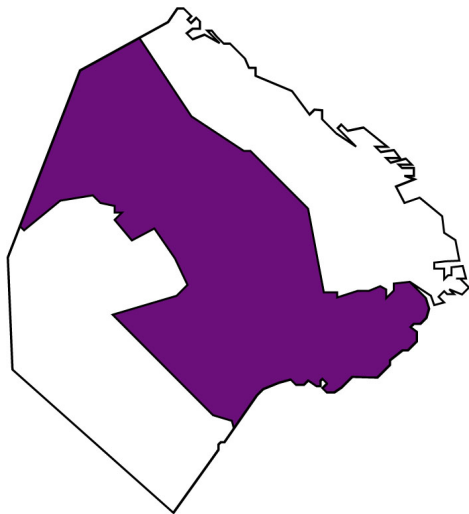
# Received bids

Telecom
99 schools
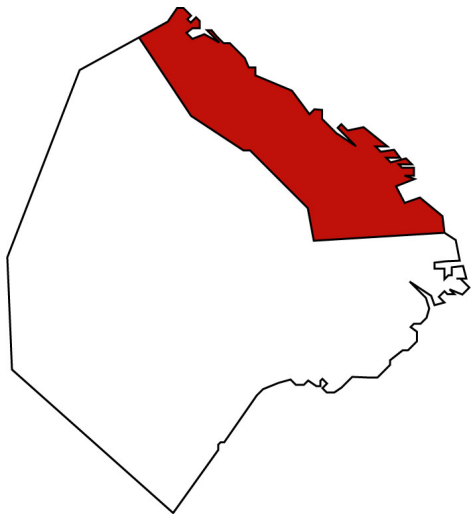
# Received bids

Telmex
97 schools
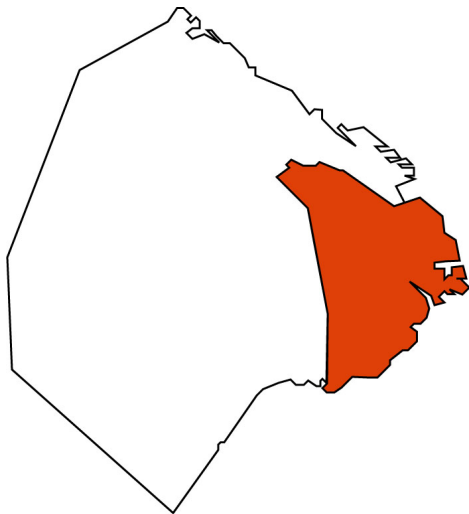
# Territorial units/regions defined by the bids
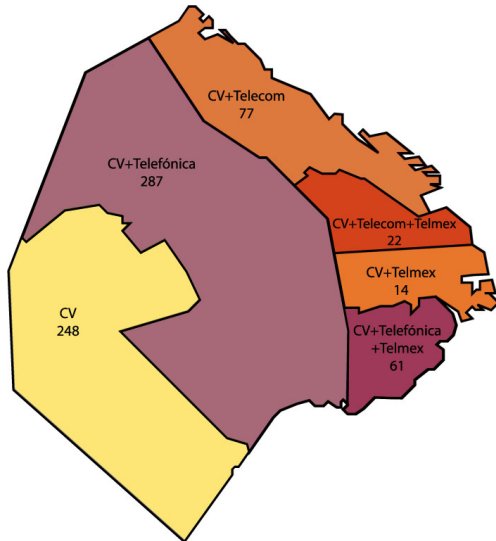
# Bids: Individual price as a function of the price interval

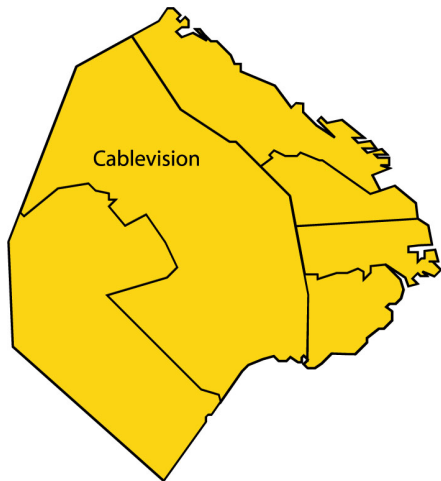| Tramo | Cablevision | | Telefónica | | Telecom | | Telmex | |
|---|---|---|---|---|---|---|---|---|
| | Descuento | Precio unitario | Descuento | Precio unitario | Descuento | Precio unitario | Descuento | Precio unitario |
| 0 - 19 | 0% | $ 4.696,70 | 0% | $ 2.662,00 | 0% | $ 1.991,66 | 5% | $ 1.605,50 |
| 20 - 39 | 0% | $ 4.696,70 | 18% | $ 2.182,84 | 0% | $ 1.991,66 | 10% | $ 1.521,00 |
| 40 - 59 | 0% | $ 4.696,70 | 28% | $ 1.916,64 | 0% | $ 1.991,66 | 20% | $ 1.352,00 |
| 60 - 79 | 0% | $ 4.696,70 | 32% | $ 1.810,16 | 33% | $ 1.075,50 | 25% | $ 1.267,50 |
| 80 - 99 | 0% | $ 4.696,70 | 40% | $ 1.597,20 | 45.02% | $ 890,07 | 33% | $ 1.132,30 |
| 100 - 149 | 10% | $ 4.227,03 | 50% | $ 1.331,00 | — | — | — | — |
| 150 - 199 | 15% | $ 3.992,20 | 59% | $ 1.091,42 | — | — | — | — |
| 200 - 299 | 20% | $ 3.757,36 | 61% | $ 1.038,18 | — | — | — | — |
| 300 - 399 | 30% | $ 3.287,69 | 70.5% | $ 785,29 | — | — | — | — |
| 400 - 499 | 40% | $ 2.818,02 | — | — | — | — | — | — |
| 500 - 599 | 50% | $ 2.348,36 | — | — | — | — | — | — |
| 600 - 699 | 60% | $ 1.878,68 | — | — | — | — | — | — |
| 700 - 709 | 80% | $ 939,34 | — | — | — | — | — | — |

- The prices of Cablevisión and Telmex are multiplied by 0.9 since these companies provide connections based on fiber optics (such calculation was prespecified in the auction terms). Telefónica and Telecom provide connections based on copper wires.

# Bids: Individual price as a function of the price interval
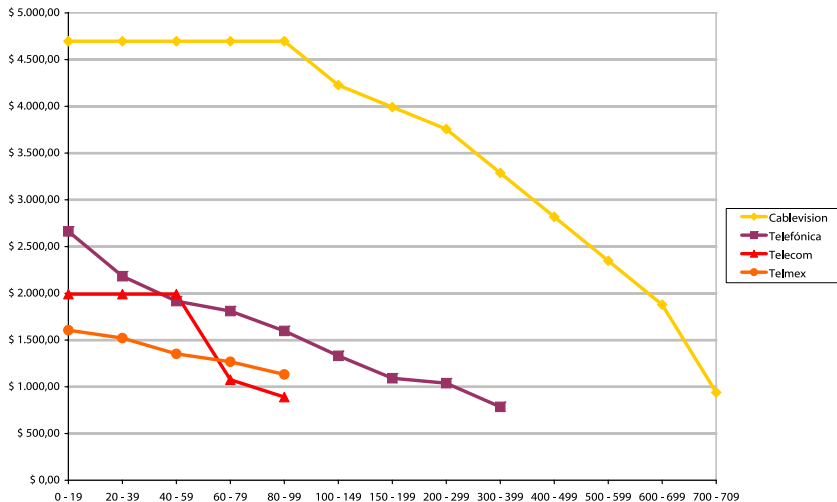
# Results

- Cablevision wins all 709 schools
- Total cost: $ 665.992,06
- Average cost per school: $ 939,34

# Conclusions

- Our main contribution was the design of the auction process since, due to the number and properties of the received bids, the optimal assignment was not difficult to find.

- By analyzing the results (final price: $ 665.992,06), we can estimate the overall benefit by making reasonable conjectures on the bids that the companies would have made in a school-per-school auction.

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# Conclusions

- The cost is $ 1.539.330,21 assuming that in the schools where CV is the only bidder, this company offers its worst price ($ 4696,7), and that in the remaining schools each company offers its best price. Note that this assumption is not unlikely to hold, since CV used its monopoly to win all 709 schools with an average price which is higher than the best received price, hence it could have used this monopoly in order to charge a high price in the schools where it is the only bidder.

- Savings: $ 1.539.330,21 - $ 665.992,06 = $ 873.338,15

## Conclusions

- The cost is $ 840.461,25 making the *very optimistic* assumption that in the schools where CV is the only bidder, this company offers its second best price ($ 1878,68), and in the remaining schools each company offers its best price.

- Savings: $ 840.461,25 - $ 665.992,06 = $ 174.469,19

## Conclusions

- The cost is $ 827.931,33 making the *even more optimistic* assumption that in every school with two or more bidders, some company offers the best received price ($ 785,29) and that CV offers its second best price ($ 1878,68) in the schools where it is the only bidder.

- Savings: $ 827.931,33 - $ 665.992,06 = $ 161.939,27

- This implies that the real solution achieved a 20 % savings. Over the two years of the supply contract the global savings would have been about a million dollars.

# Conclusions

- Also revealing is what the cost would have been to the city if it had been possible to contract for each competition unit with the firm bidding the best (most discounted) price.

- For example, in the competition unit determined by CV and Telefónica, this firm would have been contracted because its best price, $ 785.29, is better than CV's.

- In fact, such assignments would not be realistic because the firms' best prices were bid for high numbers of schools which they would not necessarily be awarded in this scenario, but they do serve as a lower and very optimistic bound for what the city might have paid.

# Conclusions

- The total monthly price on this assumption would have been $ 607,507.68, which is only 9 % lower than the price actually obtained by the city.

- This is a relatively low figure and gives an idea of the strong competition between bidders the tender process was able to generate.

## Conclusions

- Some perspective on the effectiveness of the proposed tender format may be had by calculating what would have been the highest unit price CV could have bid for the most discounted volume interval and still be awarded the contract for every school.

- This can be done by running our polynomial model with different values for CV's most discounted price.

- A binary search was done over the range of possible values for this price until the limit value was found.

- The highest price turned out to be $ 1605.52, or 71 % above the actual bid value. This suggests that the tender was indeed competitive and there was no collusion between the bidders.